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Collaterization of Global Assets and Stochastic Leverage

Seminar
at the
Higher School of Finance and Management of the ANE
May 15, 2014
Moscow

Methodological Remarks

The model should be simple:
as simple as possible, but no simpler
Albert Einstein

There are two kinds of forecasters:
those who don't know,
and those who don't know they don't know
John Kenneth Galbraith

Presentation Structure

Leverage and Financial Instability
(*Voprosy Economiki*, #9, 2012)

Logistic Model of Financial Leverage
(HSE Economic Journal,vol.17, #4, 2013)

Financial Assets Collaterization and Stochastic Leverage
(HSE Economic Journal,vol.18, #2, 2014)

The Papers Overview:

1. The Problem Formulation
2. The Logistic Model of Leverage
3. The Model Microfoundations
4. Wicksellian Analysis of Macrofinance
5. Stochastic Leverage Dynamics
6. Collaterization of Global Assets
7. Some Conclusions

Assets Collaterization Problem Overview

1. The Problem Formulation

Crises:

Intra-systemic approach – models of percolation (2007-11)
Supra-systemic approach – logistic models (2012-14)

K. Wicksell, I. Fisher, J. K Galbraith, J.M. Keynes, B. Mandelbrot, L. von Mises, H. Minsky, J. Tobin on the idea of conjugated, coherent and contradictory development of financial and real markets.

Theoretical and Empirical Aspects of the Problem

Collaterization of Assets and the Wicksellian concept of
the Natural Rate of Interest

Leverage and collaterized loans with margin calls
W. Shakespeare, *The Merchant of Venice*

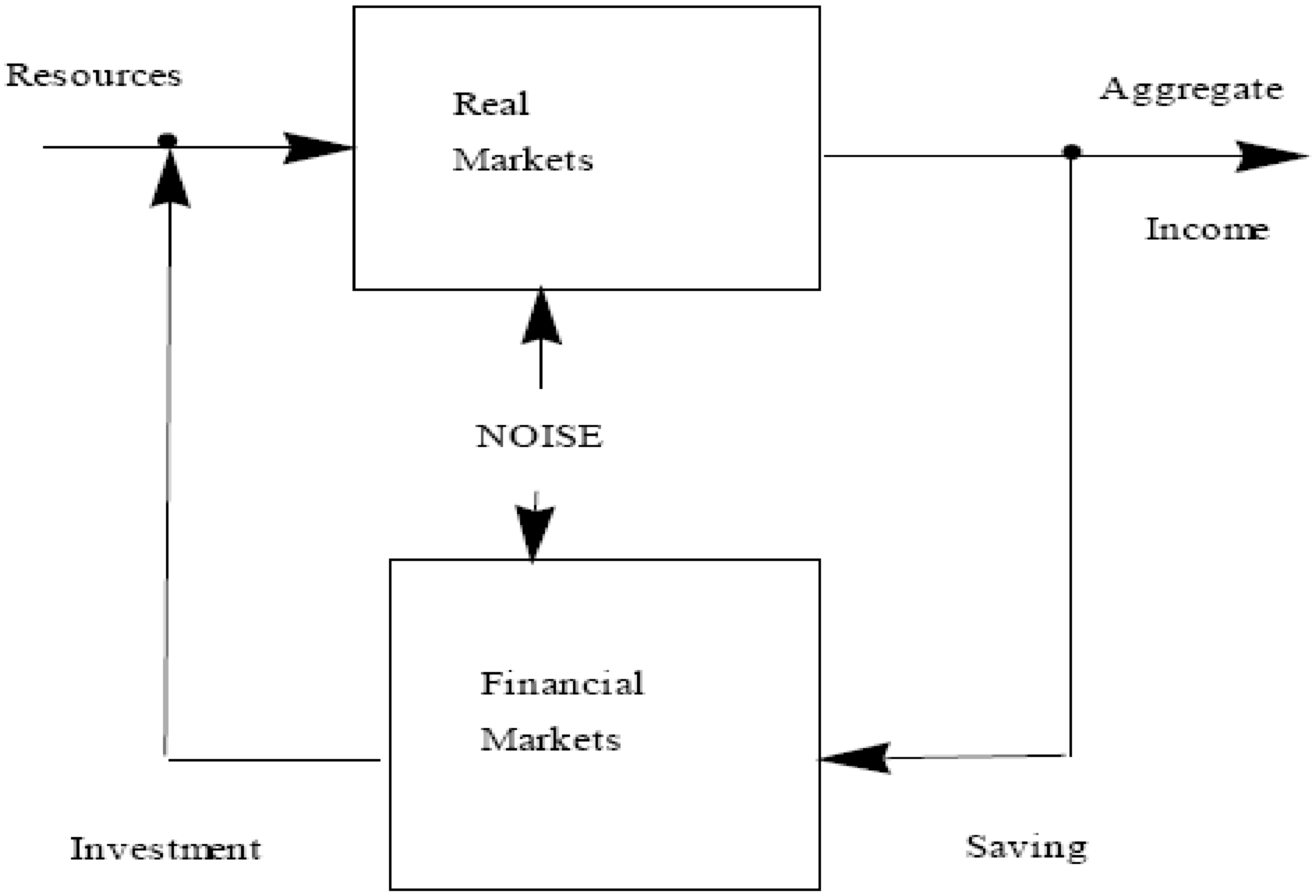
The logistic model:
Th. Malthus hypothesis and ***P.-F. Verchulst*** equation

BIS and the **Fed** on the leverage monitoring and control

The IMF, *Global Financial Stability Reports*

The Problem: **Does a stable measure of macrofinancial assets collaterization exist in the long and short run? Whether is it possible to make such estimations for the Global Financial System?**

Financial and Real Markets



Logistic Model in Finance: General Approach

2. The logistic model of leverage

The leverage differential, $dl(t)$, and its level, $l(t)$ are connected

$$dl = f[l(t)]dt,$$

where both functions depend parametrically upon time.

RHS can be decomposed to represent *the coupling*
(in terms of the complex systems theory)
between the leverage and its rate of change

$$f[l(t)] = l(t) * g[l(t)].$$

The Verchulst Equation in Finance

A Taylor series expansion at $l(t) = l^*$, where $g(l^*) = 0$:

$$g(l) \cong g(l^*) + g'(l^*)(l - l^*) = -g'(l^*)l^* + g'(l^*)l = a - bl \,.$$

leads to the **logistic model of leverage dynamics**:

$$dl_t = l_t(a - bl_t)dt \,,$$

where $a = -g'(l^*)l^*$ и $b = -g'(l^*)$.

Yet this approach is too general to provide a convincing description of the financial process.

MacroFinancial Balances and Flows

The balance of the three ***state variables*** of an aggregate financial system:

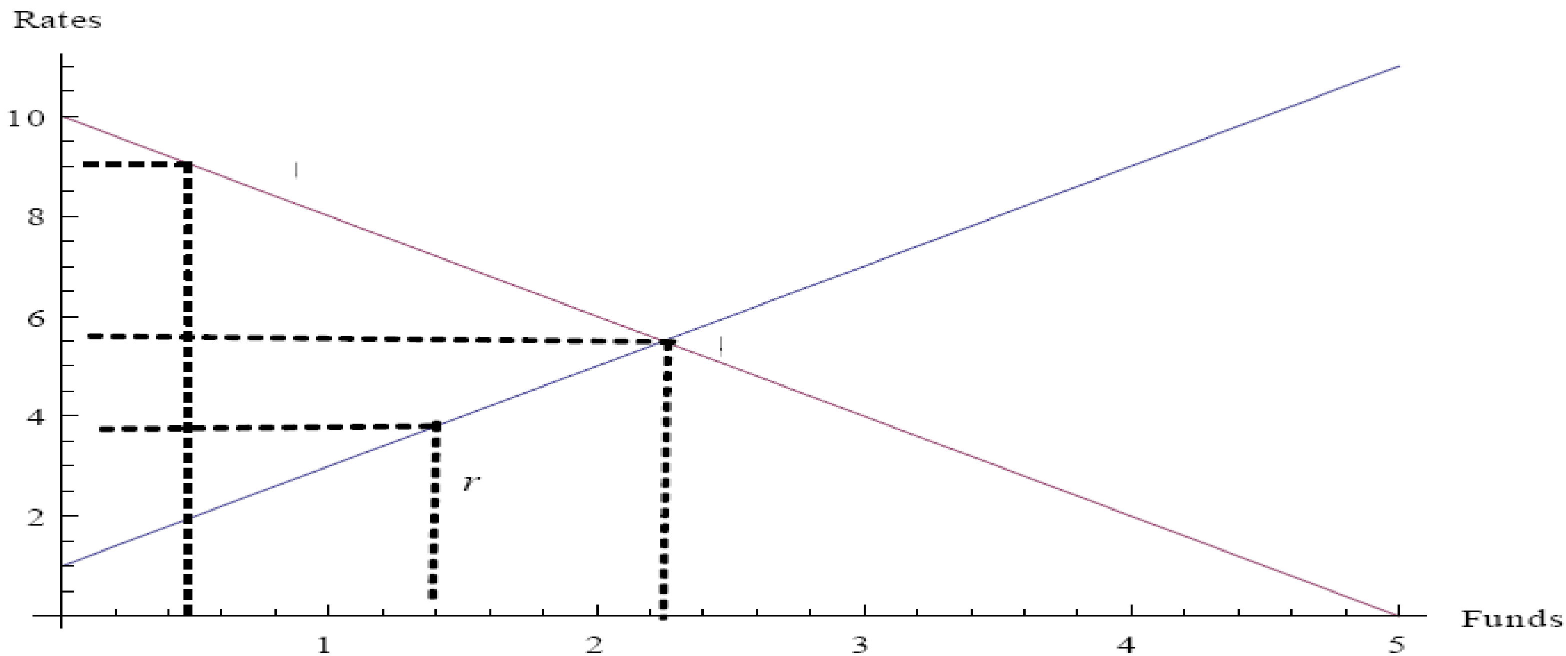
$$A(t) = x(t) + e(t).$$

The balance of **financial flows**:

Aggregate Saving = Aggregate Debt + Aggregate Investment

$$dA(t) = dx(t) + de(t).$$

MacroFinancial Saving and Investment



Yields: $ROE \equiv \rho; \text{ and } de(t) = \rho e(t)dt;,$

$ROA \equiv \mu; \text{ and } dA(t) = \mu A(t)dt$, refinancing rate, $r > 0$ $dx(t) = rx(t)dt$.

Global Financial System Empirical Data for 2004-12

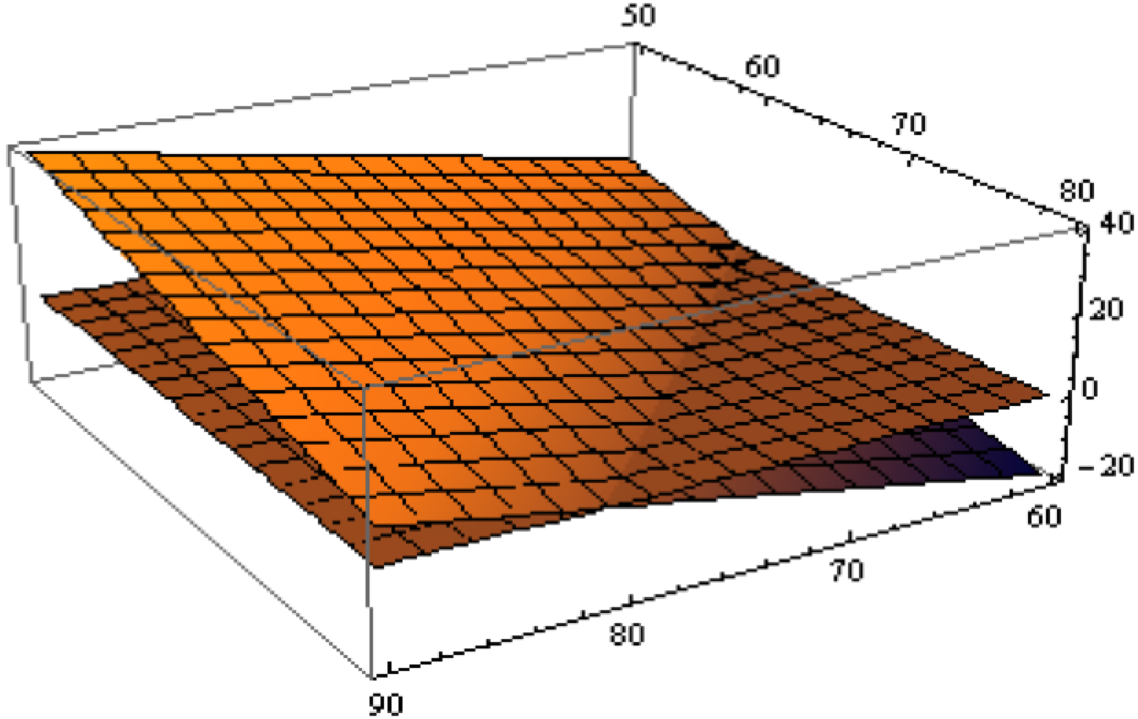
Changes in global assets, debts and capital in $ tn (GFSR, 2003-13)

	2004	**2005**	**2006**	**2007**	**2008**	**2009**	**2010**	**2011**	**2012**
ΔA_t	16.4	7.1	38.6	39.3	-15.3	17.8	17.9	5.8	12.7
Δe_t	6.0	0	13.6	14.3	-31.6	13.6	7.9	-8.0	5.4
Δx_t	10.4	7.1	25.0	25.0	16.3	4.2	10.0	13.8	7.3

Given the ***Global Capital of $ 65.1 tn in 2007*** the following system

$$\begin{pmatrix} A \\ x \end{pmatrix} = \begin{pmatrix} -0.0666 & -0.0990 \\ 1 & -1 \end{pmatrix}^{-1} \begin{pmatrix} -31.6 \\ 65.1 \end{pmatrix}$$

has a solution of $229.7 tn (Global Assets) and $164.6 tn (Global Debt) which coinsides with empirical volumes for 2007 year.



Financial Flows and Leverage

Static relations among the rates of financial flows and **the leverage**:

$$l(t) = A(t) / e(t),$$

are connected for $1 \le l < \infty$, **and** define the equation of
the balanced financial market :

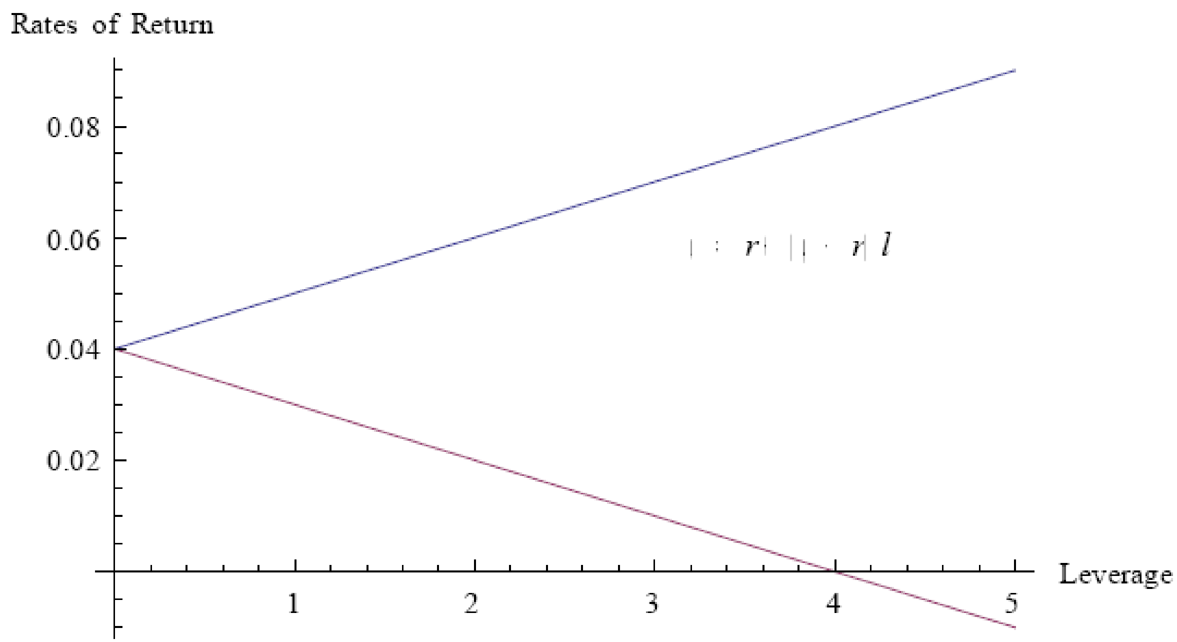
$$\rho = r + (\mu - r)l,$$

or $$\rho = \mu + (\mu - r)[l - 1]$$

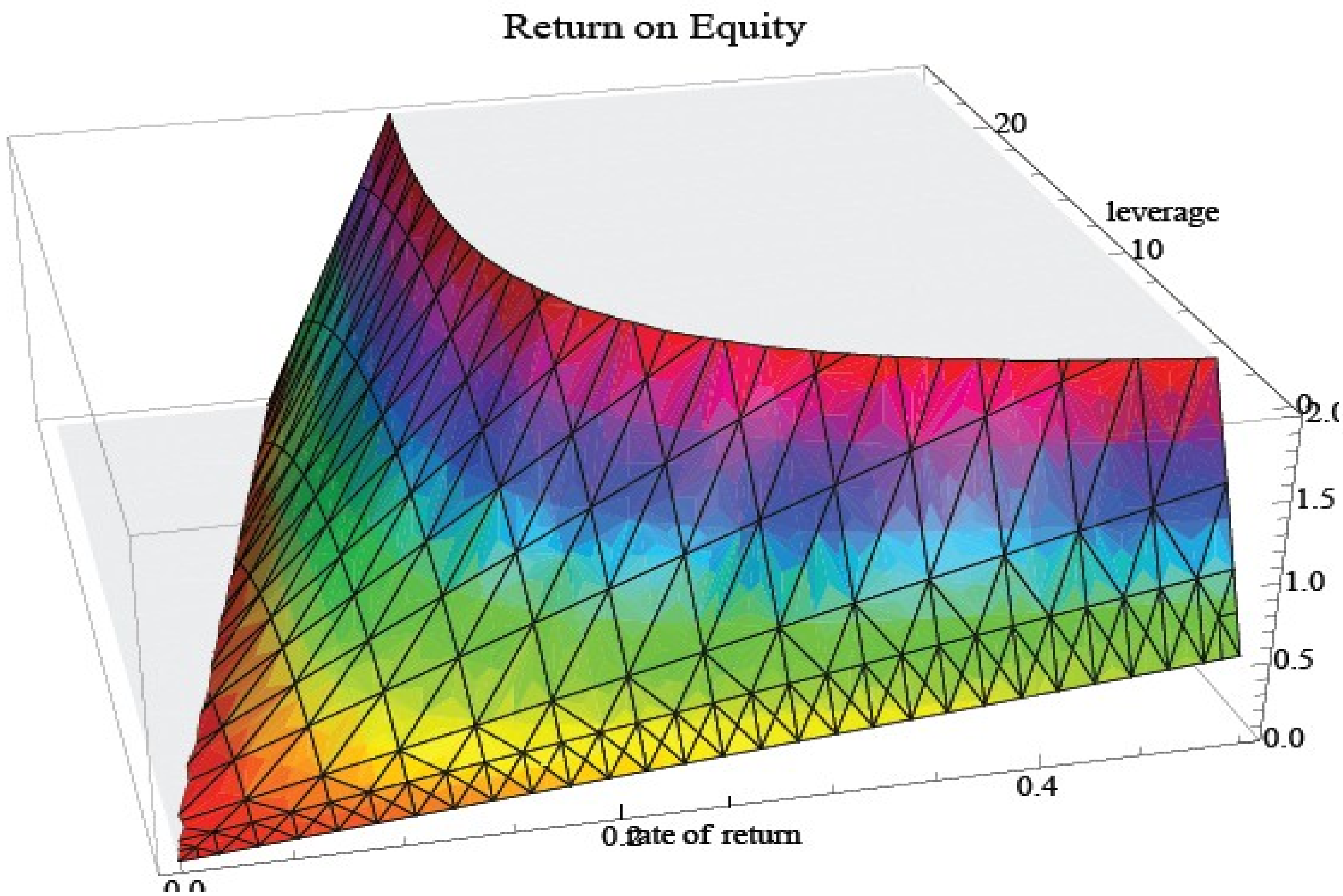
for positive and negative spreads $(\mu - r)$.

Yields and Leverage Equations

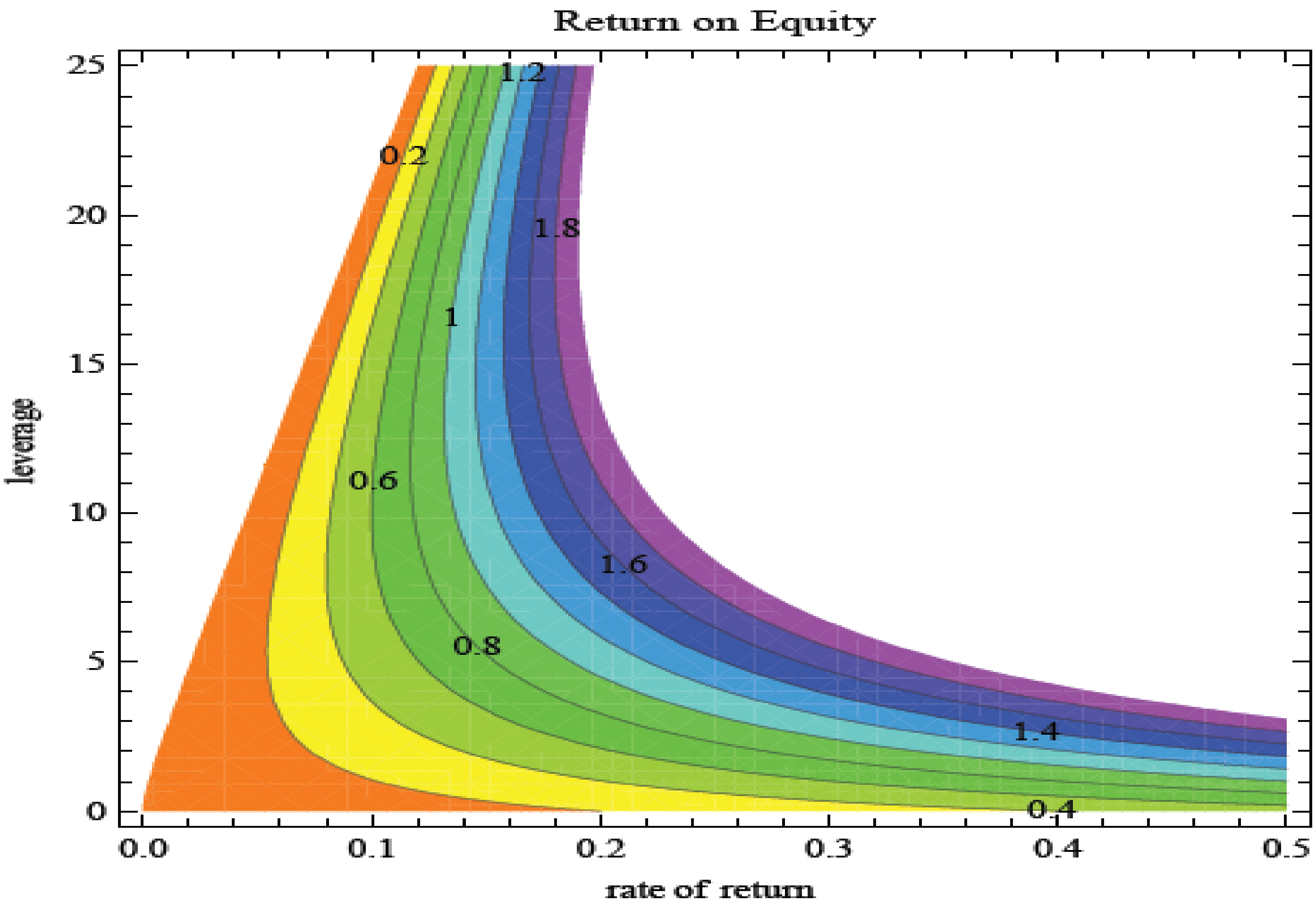
Hence there are **two conjugate regimes** for the "normal" and the "irrational" balanced market.



The ROE surface for the feedback dynamics of leverage



Isoquants of the ROE surface



A Mergers&Acquisitions Deal

3. The Model MicroFoundations

Given two companies, L and U, with the same assets $A_U = A_L = A$ and generating the same return ΔA

	t	T
Buy company L	$-e_L = e$	$+\rho e_L = \rho e$
Buy company U	$-e_U = -A$	$+\rho_U e_U = \mu A$
borrow	$+x$	$-r x$
	$-e = -A + x$ $e = A - x$	$\rho e = \mu A - r x$ $\rho = r + (\mu - r) l$

If credit equals the debt of the first company, then the ***MM Proposition*** takes place.

A Particular Case of a Leverage Equation

The DuPont model:

$$\rho = \frac{net\ income}{capital} = \frac{net\ income}{sales} \times \frac{sales}{assets} \times \frac{assets}{capital} \quad \text{or} \quad \rho = \frac{\Delta A}{A} \times \frac{A}{e} = \mu l\,,$$

is a particular case of a leverage equation, for $r = 0$.

Leverage Dynamics with a Feedback

The proposed methodology differs from the analysis of collaterized loans by ***J. Geanokoplos*** and the leverage by ***Shin***.

It is assumed that **leverage dynamics** follow differential equation

$$dl(t) = (\mu - \rho) l(t) dt \, .$$

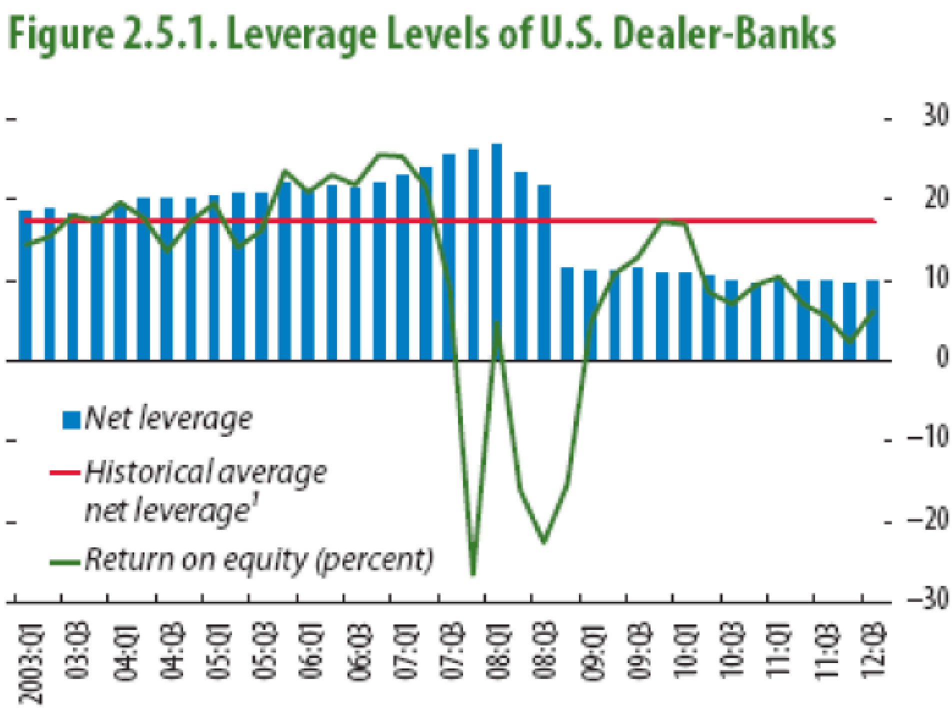
Being added with a feedback loop:

$$\mu - \rho = (\mu - r)[1 - l] \, ,$$

it forms **the logistic leverage model**:

$$dl(t) = (\mu - r)[1 - \frac{l(t)}{l^*}] l(t) dt; \; l(0) = l_0 \, .$$

ROE and Leverage for Five Largest BHCs



Source: Company reports.
[1]For the period shown.

Leverage Logistic Model

The alternative representation of a **leverage model:**

$$dl(t) = a[1 - \frac{1}{K} l(t)] l(t) dt \equiv [a - bl(t)] l(t) dt$$

where $l^* \equiv K = a/b = \frac{\rho - r}{\mu - r}$ is the stationary leverage;

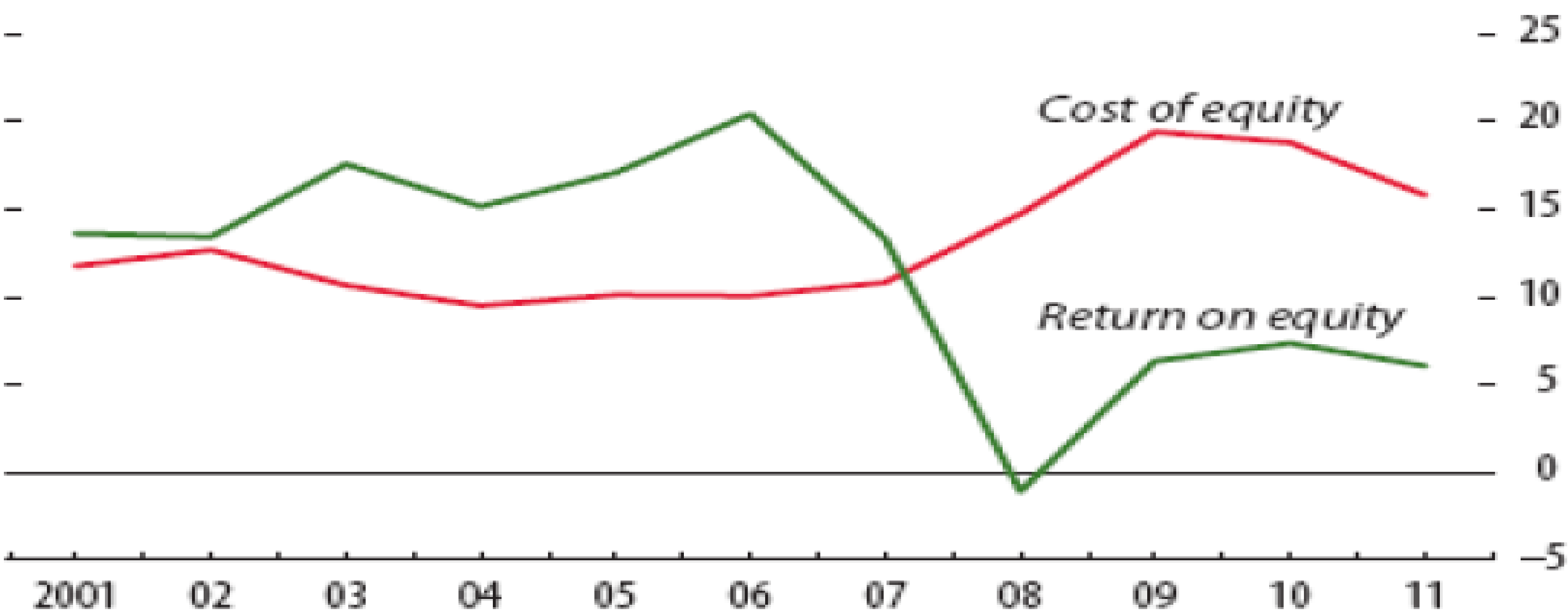
$a = \mu - r$ is the WACC/refinancing (deposit) rate;

$aK = \rho - r$ is the spread ROE/refinancing (deposit) rate;

$b = \frac{(\mu - r)^2}{\rho - r}$ is the drag parameter.

Empirical Cost and Return on Equity

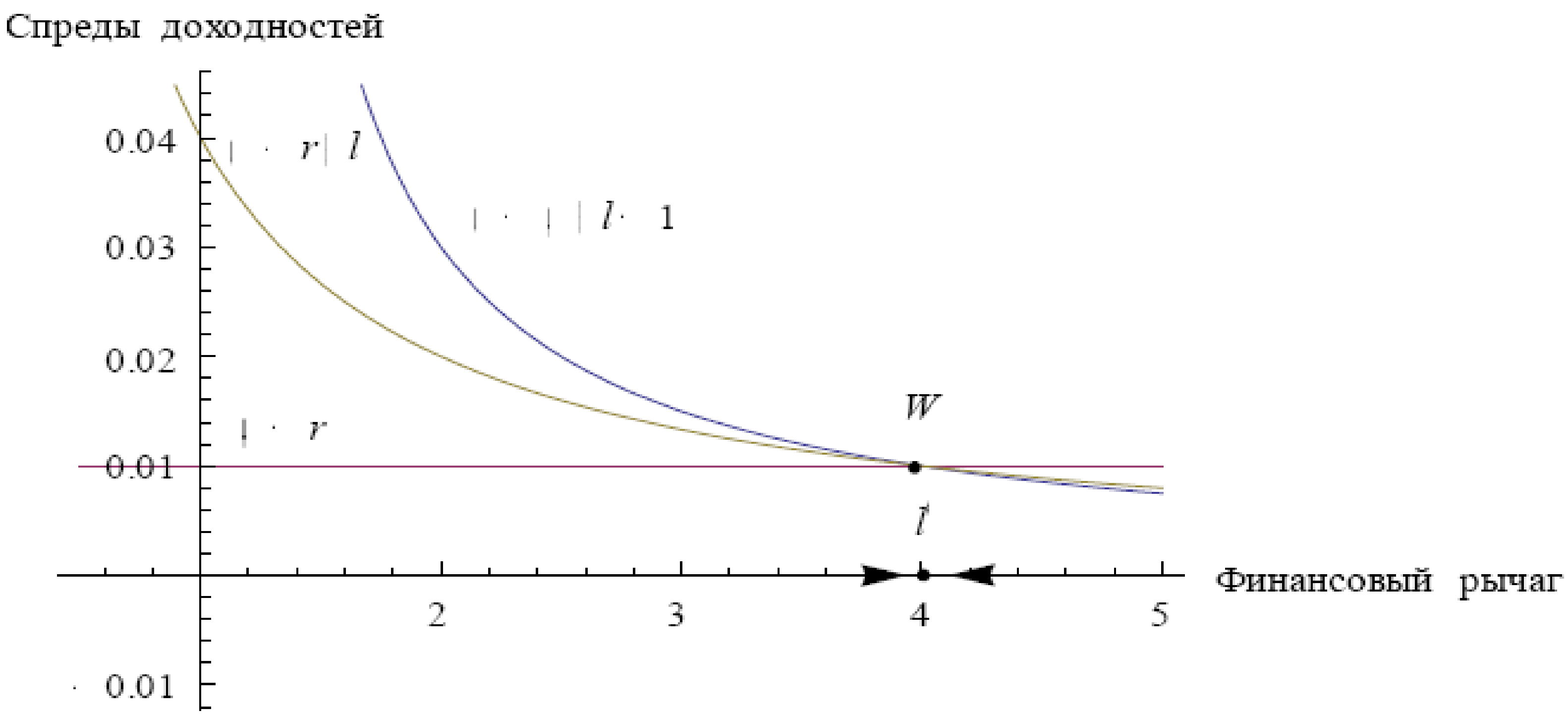
Figure 2.5.3. Return on Equity versus Cost of Equity for All U.S. Dealer-Banks
(In percent)



Sources: Bloomberg L.P.; and SNL Financial.

The Local Dynamics of Leverage I

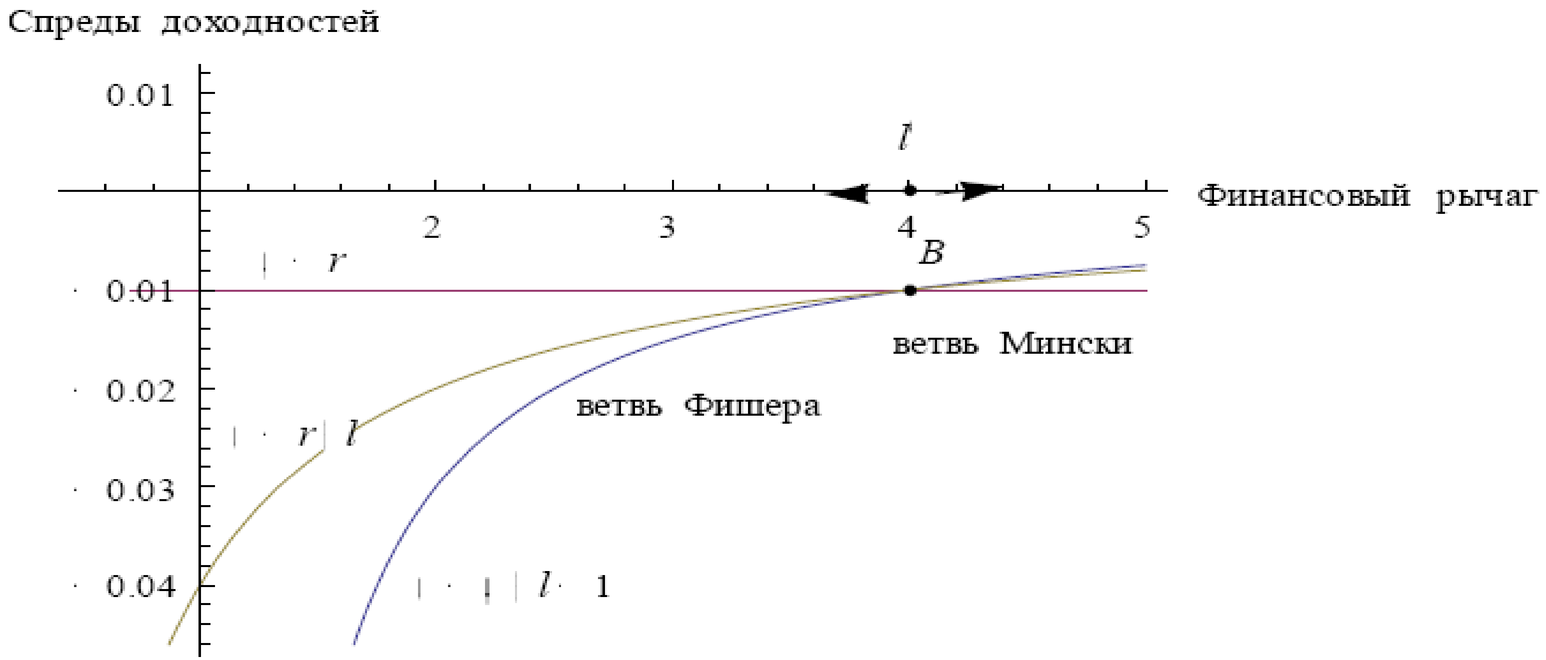
4. Investors' behavior near stationary Wicksellian point W



The Local Dynamics of Leverage II

Investors' behavior near unstable stationary point.

The Minsky and the Fisher markets: $a < 0$.



Local Behavior of Leverage

Wicksellian Rules of Investors' Behavior

	$a=\mu-r$	$\rho-r$	$b=a/K$	$l^{*}\equiv K$	$l=3$	$l=5$
Point W	0.01	0.04	0.0025	4	$0.013>0.01$	$0.008<0.01$
Point B	-0.01	-0.04	-0.0025	4	$-0.0013<-0.01$ $0.013>0.01$	$-0.008>-0.01$ $0.008<0.01$

$$positive\ yield > positive\ \cos t \Rightarrow borrow \Rightarrow leverage \uparrow$$
$$positive\ yield < positive\ \cos t \Rightarrow sell-off \Rightarrow leverage \downarrow$$

$$negative\ yield > negative\ \cos t \Rightarrow loss < economy \Rightarrow buy$$
$$negative\ yield < negative\ \cos t \Rightarrow loss > economy \Rightarrow sell$$

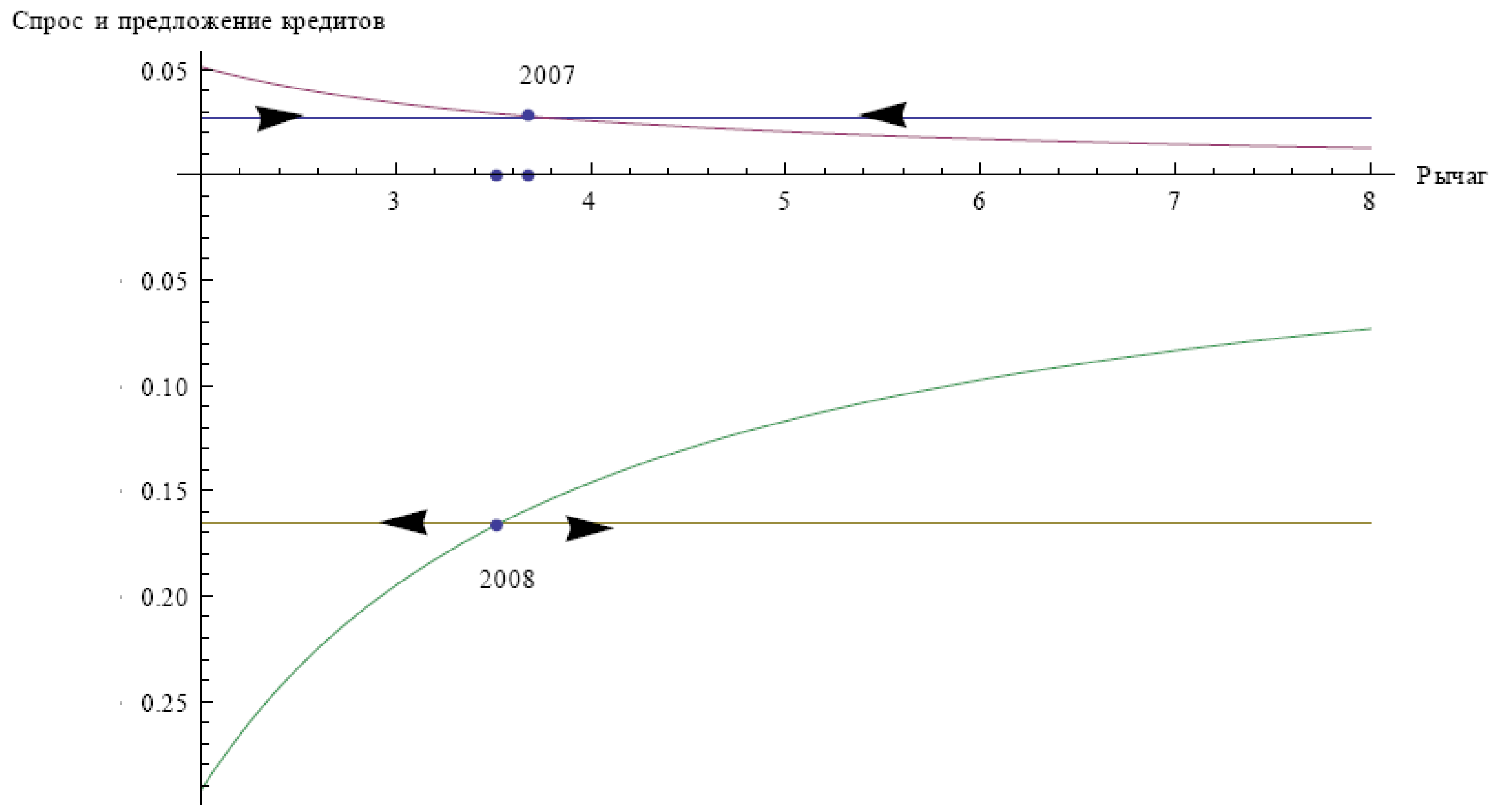
Empirical Verification of the Logistic Model

Credit Crunch 2007- 08 Parameters

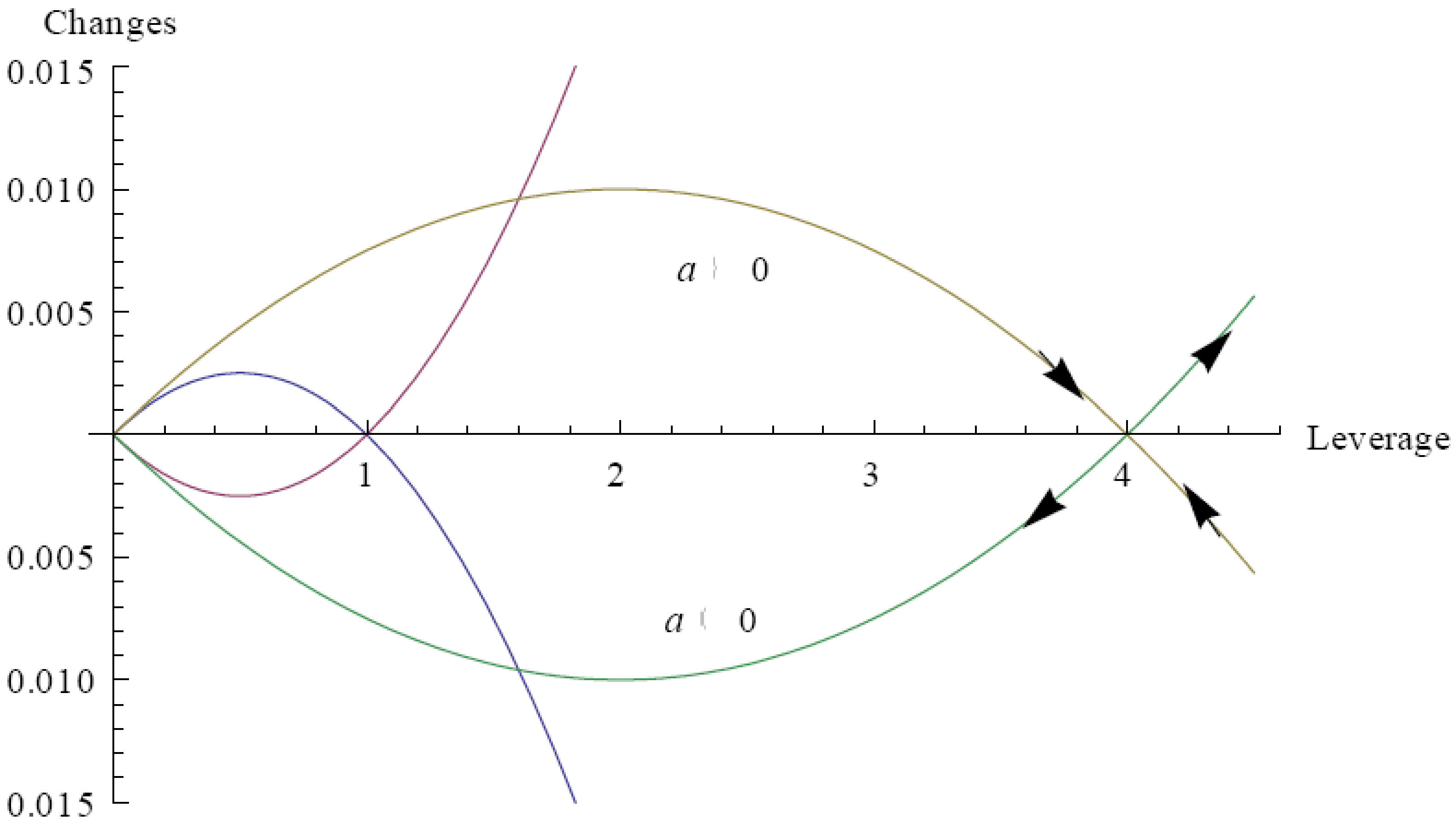
	ρ	μ	r	$a=\mu-r$	$c=\rho-r$	$b=\frac{(\mu-r)^2}{\rho-r}$	K
2007	0.2805	0.2064	0.1791	0.0273	0.1024	0.00728	3.75
2008	–0.4854	–0.0666	0.0990	–0.1656	–0.5844	–0.04691	3.53

Logistic Model: Empirical Verification

Attractor and Repeller of the Global Finance in 2007-08



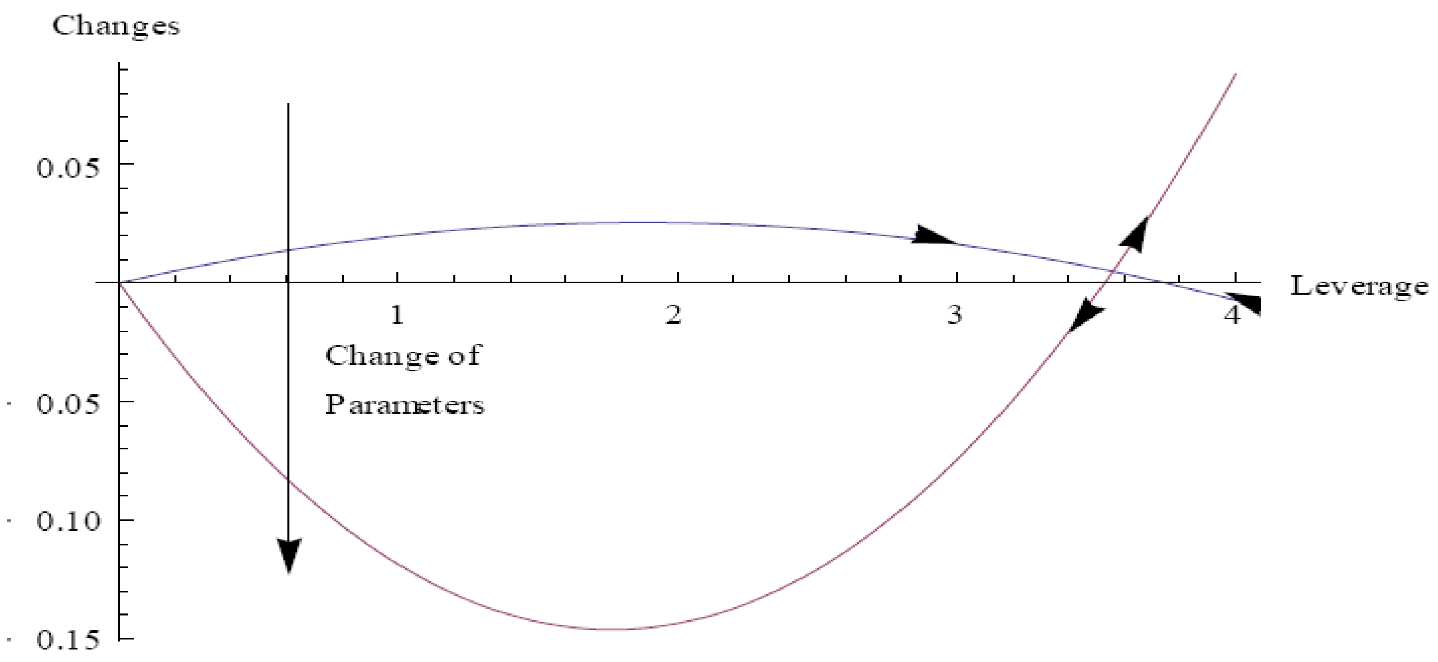
Financial Market Phase Diagram



Credit Crunch 2007-08 Phase Portrait

The phase diagram of the leverage dynamics showed that

the **system bifurcates at the origin** $(\mu - r) = 0$



Solution to the Logistic Equation

Trajectories (in time) of the logistic equation solution are given as

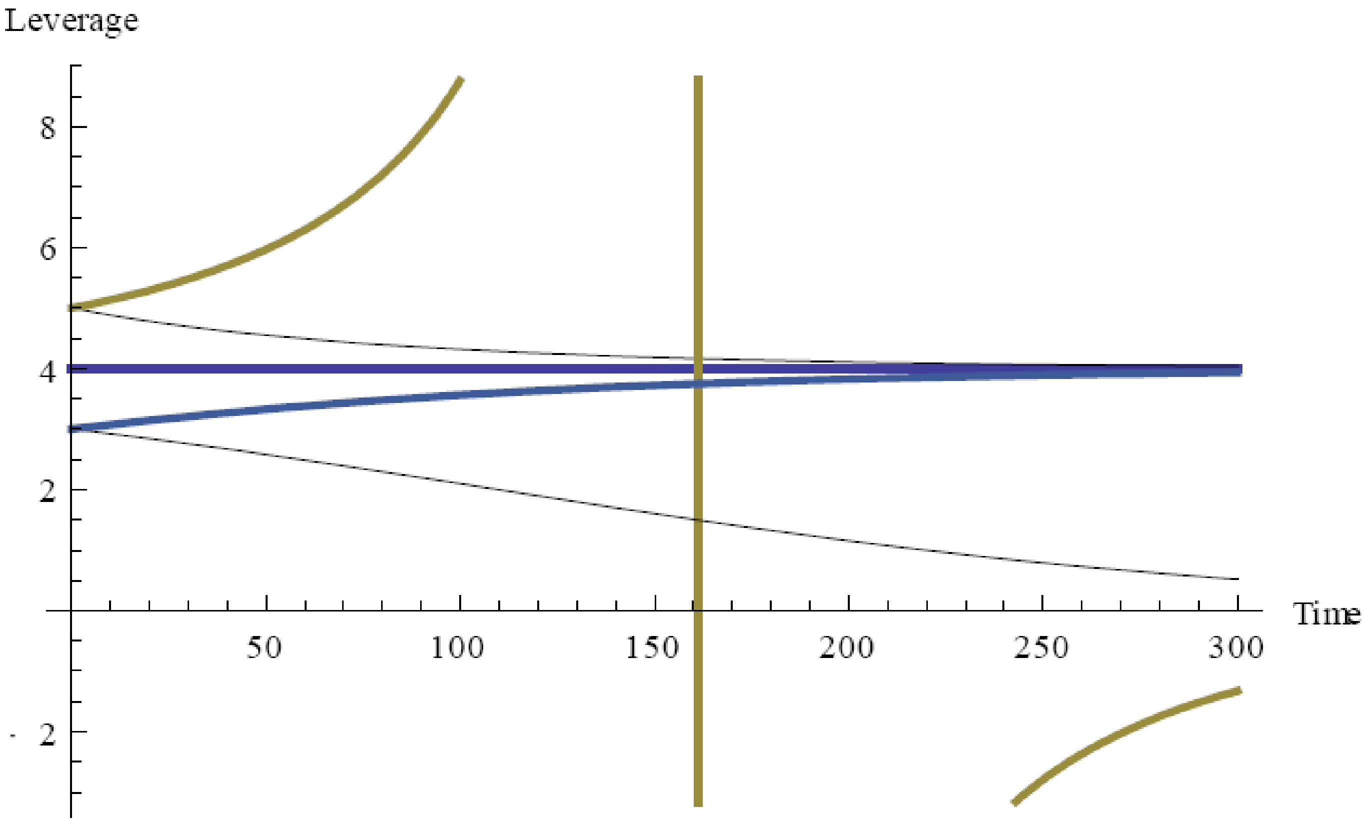
$$l(t) = l^*\{1 + (\frac{l^*}{l_0} - 1)\exp[-(\mu - r)t]\}^{-1}.$$

Solution to the logistic equation is **the weighted harmonic average**

$$l(t) = \frac{l^* l_0}{l_0(1 - \exp[-at]) + l^* \exp[-at]} \quad \text{where} \quad a = \mu - r.$$

of the initial leverage, l_0, and its stationary state, l^*.

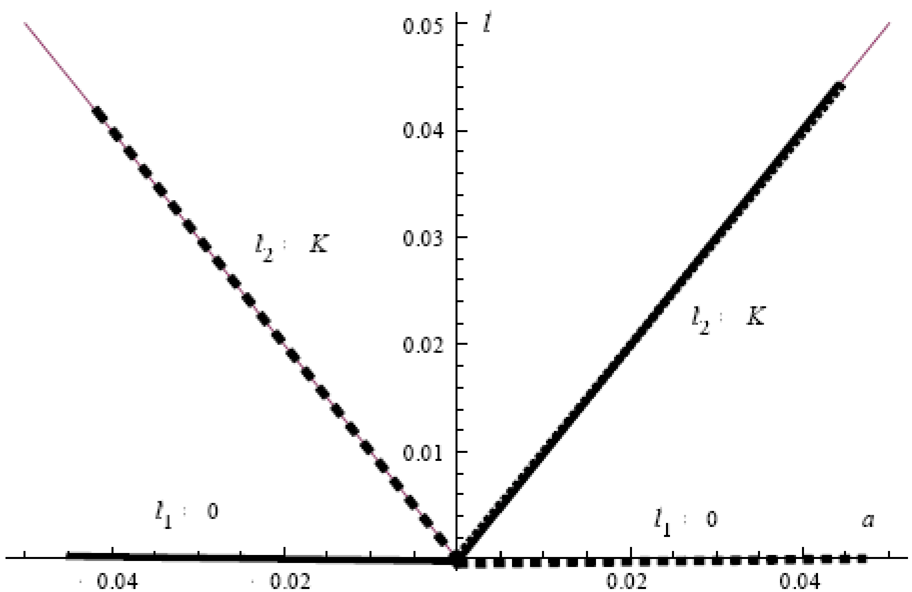
Financial Leverage Trajectories



Financial Leverage Asymptotics

States of the Market	$l^* > l_0$	$l^* < l_0$
$\mu - r > 0$	*credit stabilization,* $l_0 \to l^*$	*credit stabilization,* $l_0 \to l^*$
$\mu - r < 0$	*deleveraging,* $l_0 \to 0$	*credit* exp*ansion,* $l_0 \to \infty$

Bifurcation diagram of credit market



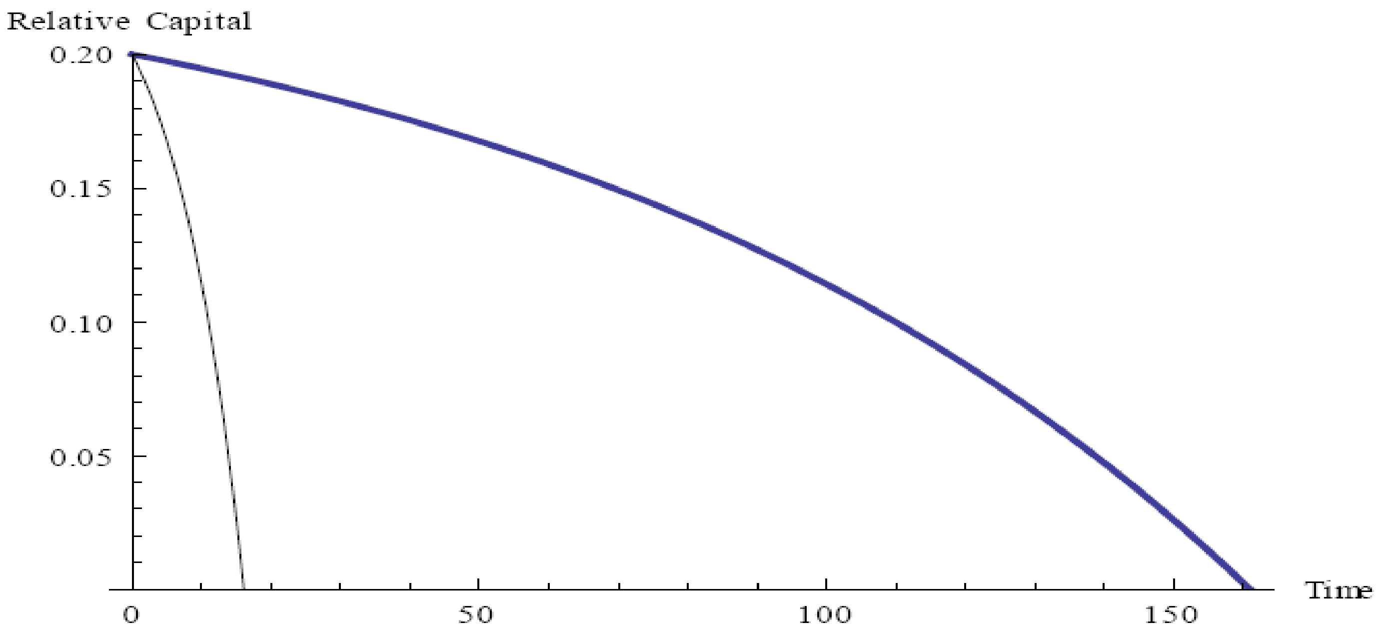
Capital Ratio Analysis

The capital ratio dynamics

The **weighted arithmetic average** of the stationary, w^*, and the initial, w_0, capital ratios:

$$w(t) = w^*(1 - \exp[-(\mu - r)t]) + w_0 \exp[-(\mu - r)t].$$

Capital Ratio Dynamics



$$w^*(1-\exp[-(\mu-r)t]) + w_0 \exp[-(\mu-r)t] = 0$$

The expected time of the credit market collapse:

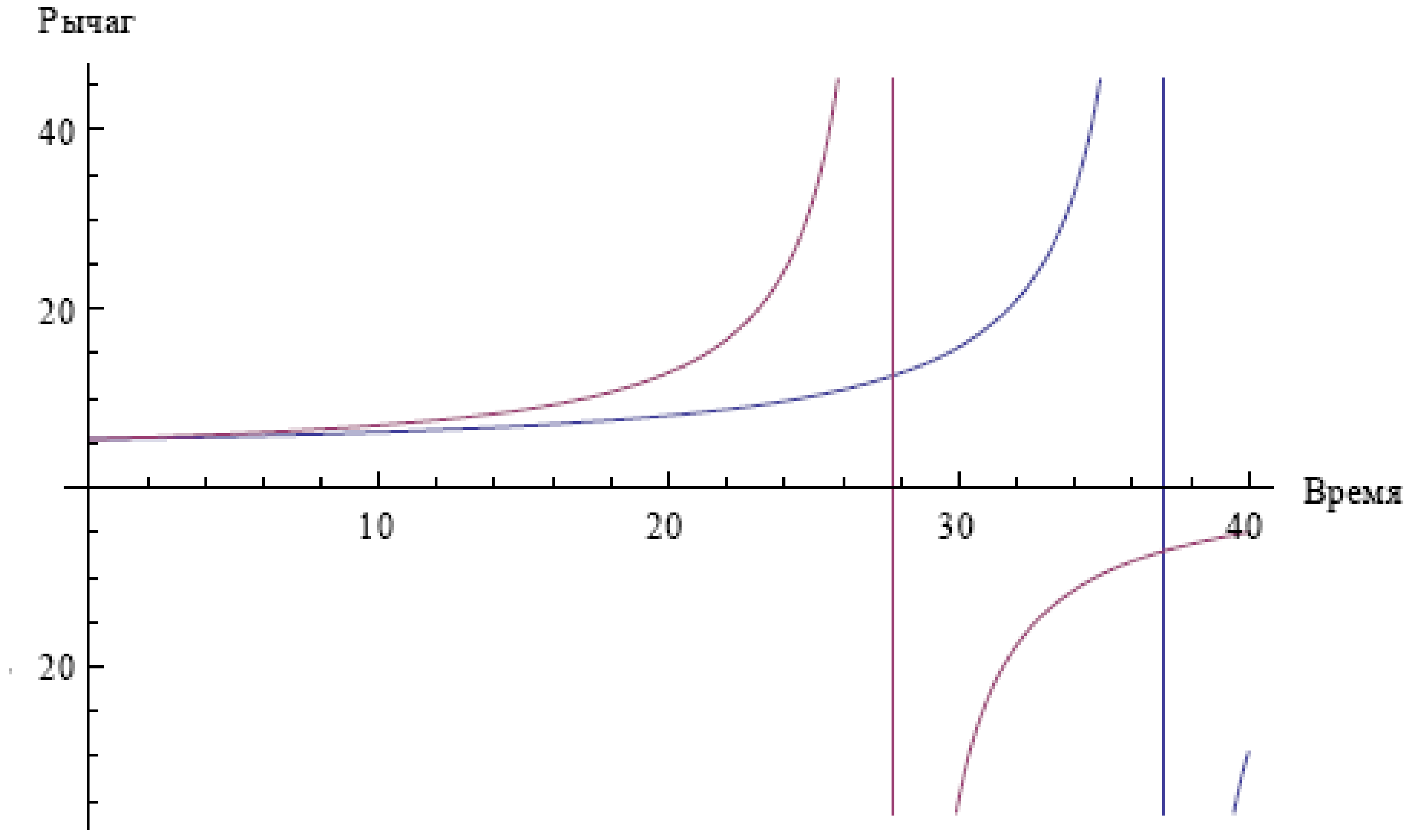
$$t_c = -\frac{1}{a}\log\frac{w^*}{w^* - w_0}.$$

Empirical data for the global leverage model

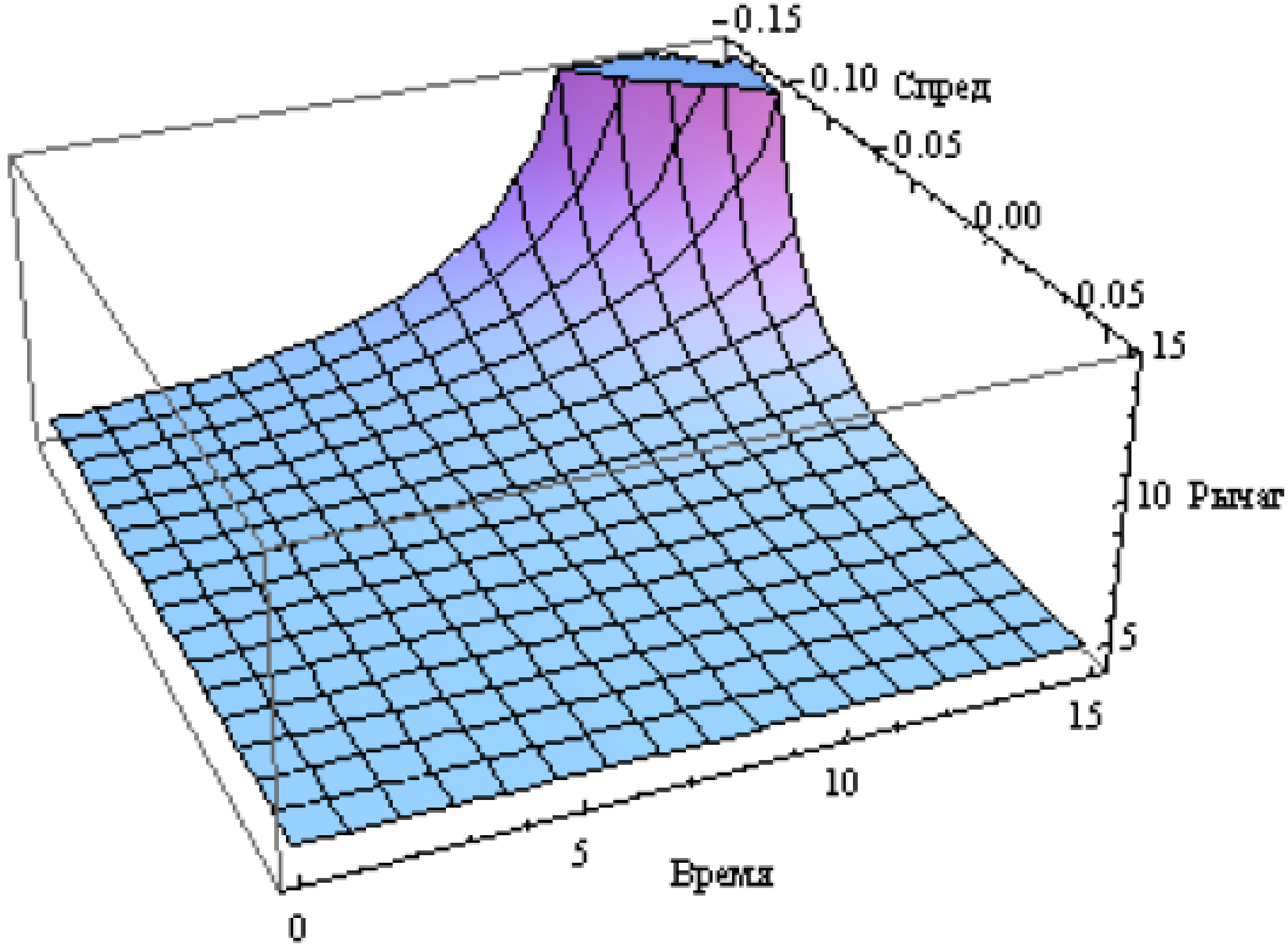
Годы	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012
μ_t		0.1278	0.0491	0.2543	0.2064	–0.0666	0.0830	0.0771	0.0232	0.0496
ρ_t		0.1923	0	0.3656	0.2815	–0.4854	0.4090	0.1674	–0.1452	0.1148
r_t		0.1071	0.0660	0.2182	0.1791	0.0990	0.0232	0.0529	0.0713	0.0350
$(\mu - r)_t$		0.0207	–0.0169	0.0361	0.0273	–0.1656	0.0598	0.0242	–0.0481	0.0146
$(\rho - r)_t$		0.0852	–0.0660	0.1474	0.1024	–0.5844	0.3858	0.1145	–0.2165	0.0747
K_{t-1}	4.12	3.9	4.08	3.75	3.53	6.45	4.73	4.50	5.46	
l_t^*	3.97	3.89	4.08	3.74	3.53	6.4	4.92	4.54	5.43	5.12

$$l(t) = 4.58\{1 + (\frac{4.58}{5.12} - 1)\exp[-a\,t]\}^{-1},$$

Possible evolution of global leverage



The surface of leverage trajectories for 2012-26



Leverage under Uncertainty

5. Stochastic Logistic Model of Leverage

Parameters of the leverage usually contain multiplicative noise:

$$dl_t = l_t[(a - bl_t)dt + \sigma\, dz_t],$$

where σ is the volatility of leverage; $z_t = \int_0^t dz_u$ is the standard

Brownian motion; $dz_t = \varepsilon_t (dt)^{0.5}$, $\varepsilon_t \sim \qquad \sigma^2)$.

The standard Brownian motion is a nonstationary process since $(dz_t)^2 = dt$.

The Soros Financial Reflexivity and Expectations

"There is a two-way reflexive connection between perception and reality which can give rise to initially self-reinforcing but eventually self-defeating boom-bust processes, or bubbles. Every bubble consists of a trend and a misconception that iteract in a reflexive manner."

{Soros, G. (2008) *The New Paradigm for Financial Markets*, Public Affairs, NY, p.X}

Stochastic Logistic Equation Solution

The "strong" solution of the stochastic logistic equation (Sciadas, 2010):

$$l(t) = \frac{l_0 K \exp[(a - 0.5\sigma^2)t + \sigma z_t]}{K + a l_0 \int_0^t \exp[(a - 0.5\sigma^2)u + \sigma z_u] du}.$$

For zero volatility, $\sigma = 0$, solutions of the deterministic and the stochastic models are coincided.

The **expected rate of growth** of the leverage is also the same:

$$\frac{1}{dt}\left\langle \frac{dL_t}{L_t} \right\rangle = a(1 - \frac{1}{K} L_t).$$

Asymptotic Behavior of Stochastic Leverage

The stationary forward Kolmogorov-Fokker-Plank equation for

$$\frac{\partial}{\partial l}[l(a-bl)p(l)]-\frac{1}{2}\frac{\partial^2}{\partial l^2}[\sigma^2 l^2 p(l)]=0$$

where $p(l)$ is the **pdf of the random leverage process** $L(t)$.

The Trivial Solution to the Stationary Equation

Solutions of the Kolmogorov equation (Pascuali, 2001)

A trivial solution:

$$p(l) = \delta(l)$$

which is a $\delta - Dirac$ distribution. It is associated with the stationary state, $l_1^* = 0$.

A non-trivial solution is a gamma-distribution:

$$p(l) = \frac{\beta^{\alpha}}{\Gamma(\alpha)} l^{\alpha-1} e^{-\beta l}.$$

It is defined for positive parameters of the shape and rate(scale),

respectively: $\alpha = \frac{2a}{\sigma^2} - 1;\ \beta = \frac{2b}{\sigma^2}$. Hence it exists for the values of :

$$0 < \sigma^2 < 2a.$$

Parameters of the random leverage

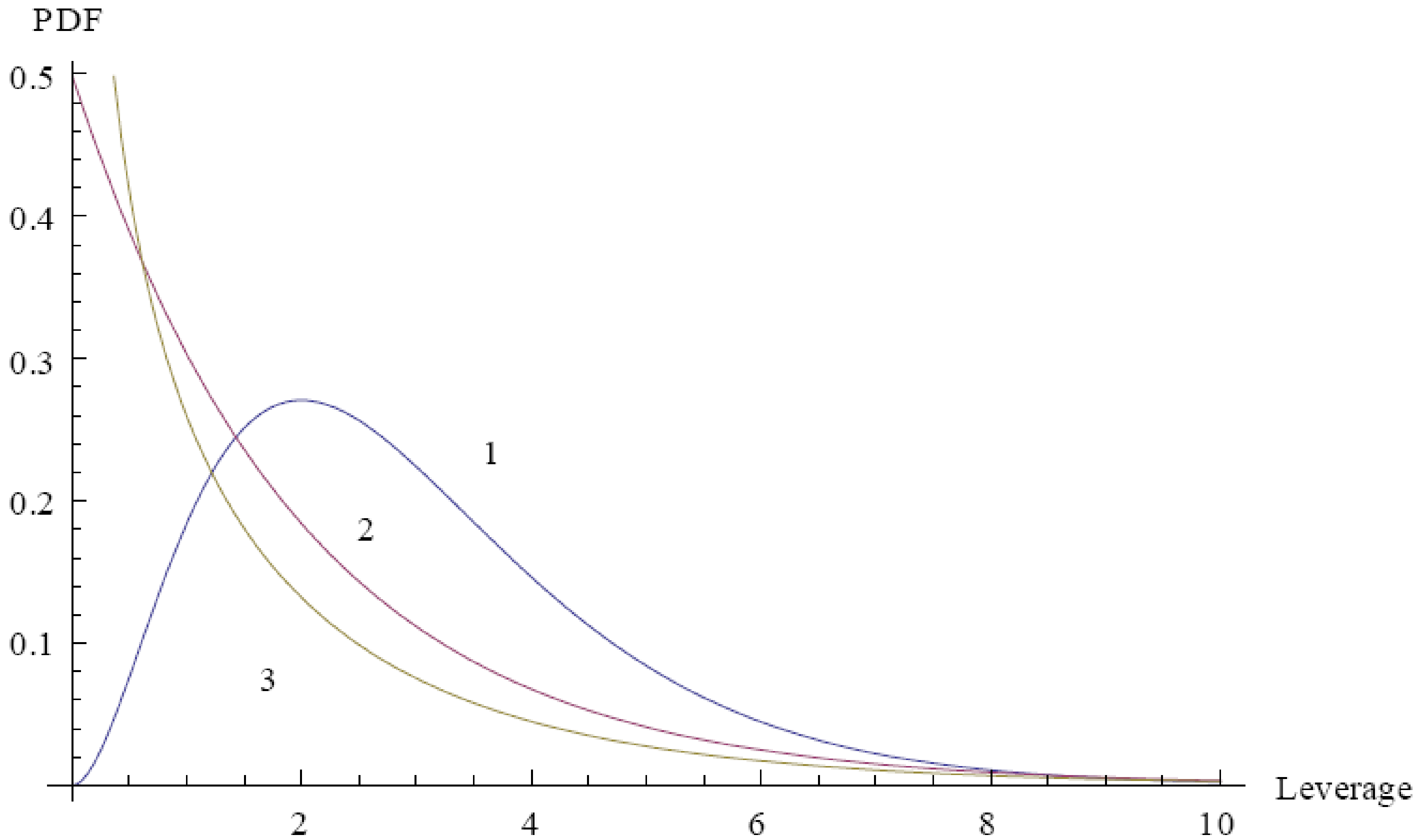
a, spread		0.04	
σ, volatility	0.14	0.2	0.22
σ^2, variance	0.02	0.04	0.05
α, shape	3	1	0.6
β, growth rate	1	0.5	0.4
$1/\beta$, scale	1	2	2.5
$\langle \lambda \rangle$ The Lyapunov exponent	-0.02	0	0.01
$\langle L \rangle$, expected value	3	2	1.5
$Mode[L]$	2	0	-

Curve 1: $\sigma^2 - a = 0.02 - 0.04 < 0$ is a peaked gamma distribution of leverage;

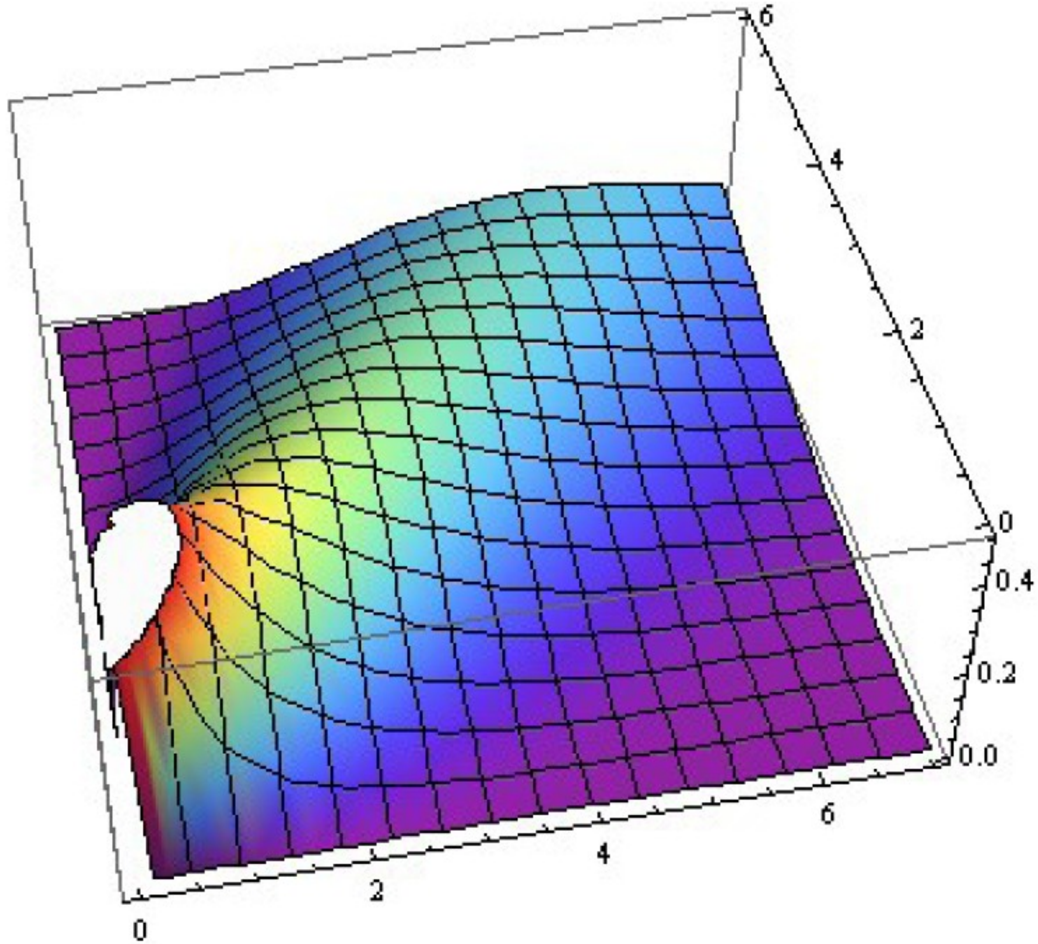
Curve 2: $\sigma^2 - a = 0.04 - 0.04 = 0$ is an exponential distribution of leverage;

Curve 3: $\sigma^2 - a = 0.05 - 0.04 > 0$ is a J-shaped gamma distribution of leverage.

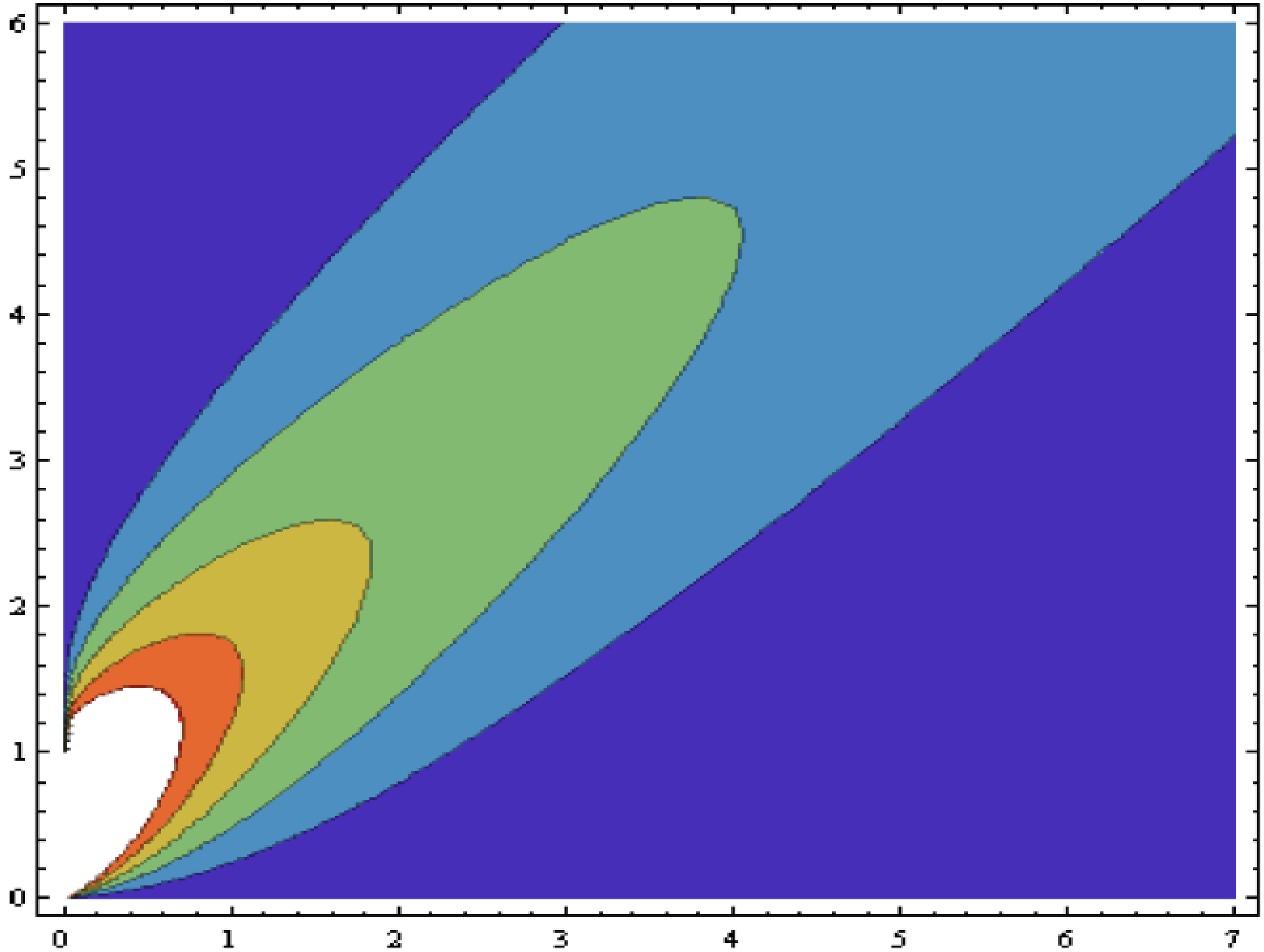
Gamma Distributed Leverage



3D Plot of a Gamma Distribution for $l \in [0,7]; \alpha \in [0,6]$



Contour Plot of a Gamma Distribution



Expectation and Mode of Leverage

The expected value of a gamma-distributed random leverage is

$$\langle L \rangle = \int_0^\infty l\, p(l) dl = \frac{\alpha}{\beta} = K - \frac{\sigma^2}{2b},$$

while its **mode** for $\alpha > 1$ is equal to

$$Mode[L] = \frac{\alpha - 1}{\beta} = K - \frac{\sigma^2}{b}.$$

The market can stay irrational longer than you remain solvent.

The Lyapunov Exponent is a measure of investors' confidence in the financial market solvency

The Lyapunov Exponent is the **expected value** of

$$\langle \lambda \rangle = \frac{\beta^{\alpha}}{\Gamma(\alpha)} \int_{0}^{\infty} (a - 2bl) l^{\alpha - 1} \exp[-\beta l] dl \, .$$

It defines stability conditions of stochastic leverage

$$\langle \lambda \rangle = \langle a - 2bL \rangle = a - 2b \langle L \rangle = a - 2b(\frac{a - \sigma^2}{b}) = \sigma^2 - a \, .$$

$$\langle \lambda \rangle = (\sigma^2 - a) \begin{Bmatrix} < \\ = \\ > \end{Bmatrix} 0; \quad \begin{matrix} \textit{stability or confidence;} \\ \textit{the system neutrality;} \\ \textit{instability or the loss of confidence.} \end{matrix}$$

Stochastic and Deterministic Models could be very different

It is important that **stability of a deterministic system does not imply stability of its stochastic analogue.** Hence the actual random leverage dynamics could differ significantly from its deterministic forecast.

Deterministic model is valid	$\sigma^2 \ll$	K
Stochastic model is stable	$\sigma^2 < a$	$K - \frac{\sigma^2}{b}$
Stochastic model is instable	$a < \sigma^2 < 2a$	$(0, K/2]$
Noise induced chaos	$\sigma^2 > 2a$	0

The Global Financial Assets Collaterization

6. Collateral Ratio for the Global Financial System

Collateral Ratio Estimation is a two-stage process:

1. Stationary (long term) parameters of gamma distribution are to be found;
2. Collateral ratio is to be estimated.

Global Financial System Leverage Dynamics in 2003-12

	2003	**2004**	**2005**	**2006**	**2007**	**2008**	**2009**	**2010**	**2011**	**2012**
l_t	3.97	3.89	4.08	3.74	3.53	6.4	4.92	4.54	5.43	5.12
$I\,l$	1.0	0.98	1.049	0.917	0.944	1.813	0.769	0.923	1.196	0.943
l_Y	3.42	3.54	3.42	3.95	4.21	3.52	4.02	3.98	3.66	3.72
$I\,l_Y$	1.0	1.035	0.966	1.155	1.065	0.836	1.142	0.990	0.920	1.016

Parameters of the Gamma Distributed Global Leverage

Stationary Gamma distribution for the Global Financial System, 2003-12

a_H	b_H	$\langle l \rangle$	$\langle l \rangle_H$	$K = a_H / b_H$	$\langle L \rangle = \alpha / \beta$	$Mode[L] = (\alpha - 1)/\beta$	σ_1^2	σ_2^2
0.034	0.0063	5.0	4.98	5.35	4.67	3.94	0.0092	0.0313

The collateral ratio for the global financial system

$$l_Y = l_t * q_t ,$$

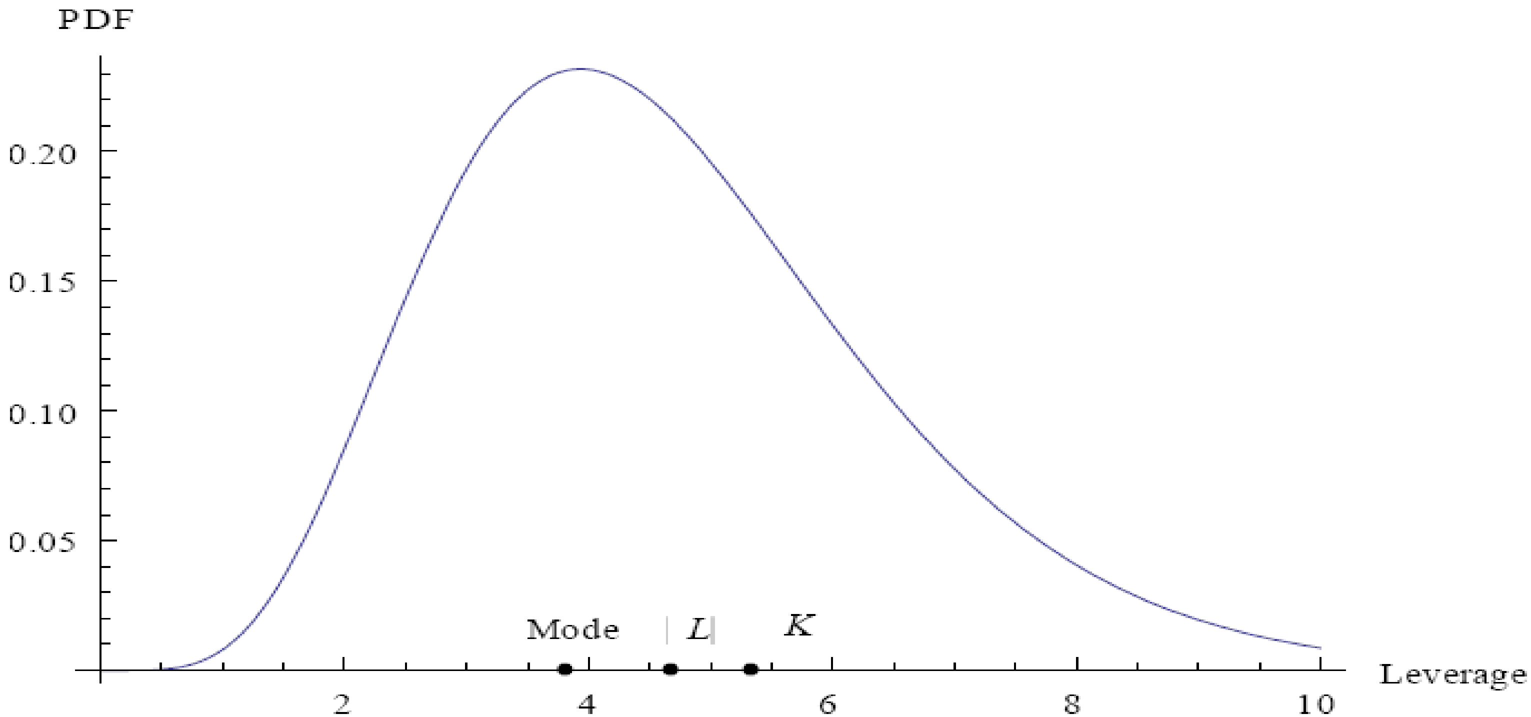
where $q_t = e_t / Y_t$ is the **Tobin's q parameter**.

$\boldsymbol{Variance\ of\ \sigma^2 = 0.0092\ scenario}$ is given by equation

$$dl(t) = 0.034[1 - 0.0063\, l(t)]\, l(t)dt + 0.0959\; l(t)dz\,,$$

and a stationary pdf function

$$p(l; 6.39, 1.37) = 0.032\, l^{5.39} \exp[-1.37\, l]\,.$$



The Global Collateral Ratio components

Годы	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012
l_Y	3.42	3.54	3.42	3.95	4.21	3.52	4.02	3.98	3.66	3.72
l_t	3.97	3.89	4.08	3.74	3.53	6.4	4.92	4.54	5.43	5.12
q_t	0.87	0.91	0.84	1.05	1.19	0.55	0.82	0.88	0.67	0.73
$l_Y = l_t * q_t$	3.45	3.54	3.42	3.93	4.2	3.52	4.03	3.99	3.64	3.74

For the leverage expectation $\langle L \rangle = 4.67$ and Tobin's parameter $q_{2012} = 0.73$ **the short run collateral ratio,** l_Y, is

$$l_Y = \frac{\alpha}{\beta} q_{2012} = 4.67 * 0.73 = 3.41 .$$

Given the World GDP in $72.2 tn, the amount of ***collaterized assets*** for the year 2012 should be equal to $246.2 tn:

$$A = l_Y * Y = 3.41 * 72.2 = 246.2 .$$

Thus about **$16.6 tn** of global financial assets (or 94 per cent) were **toxic assets.**

7. Some conclusions

a) Inertia of macrofinance, its asymptotic features.
Logistic model in the short and long run.
Logistic maps and strange attractors. Stochastic logistic model and stationary distributions;

b) An ability of the logistic model to forecast should be viewed in the context of indeterminacy of the economic proportions {the ***Reinhart-Rogoff confusion*** regarding the 90 percent threshold of debt};

c) The role of uncertainty in finance seems to be rather ambiguous. At least, it is not unilaterally negative. Since

$$Mode[L] \leq \langle L \rangle \leq K$$

the central bank is able to use the uncertainty to some extent to minimize the amplitude of a business cycle.

Thank you for your kind attention!

Global Financial Assets and GDP {IMF, GFSR 2004-12}

Динамика мирового ВВП и финансовых активов за 2003–2011 гг.

	2003 г.	2004 г.	2005 г.	2006 г.	2007 г.	2008 г.	2009 г.	2010 г.	2011 г.
Общий объем активов, трлн долл.	128,3	144,7	151,8	190,4	229,7	214,4	232,2	250,1	255,9
Темп роста	1,0	1,17	1,05	1,25	1,21	**0,93**	1,08	1,08	1,02
Рынок акций, трлн долл.	31,2	37,2	37,2	50,8	65,1	33,5	47,2	55,1	47,1
Индекс доходности акций	1,0	1,19	1,00	1,37	1,28	**0,51**	1,41	1,17	0,85
Долговые обязательства, трлн долл.	52,0	57,9	58,9	68,7	79,8	83,5	92,1	94,8	98,4
Индекс доходности долга	1,0	1,11	1,02	1,17	1,15	**1,05**	1,10	1,03	1,04
Банковские активы, трлн долл.	40,6	49,6	55,7	70,9	84,8	97,4	93,0	100,1	110,4
Темп роста активов	1,0	1,22	1,12	1,27	1,20	1,14	**0,95**	1,07	1,10
Мировой валовой внутренний продукт, трлн долл.	36,2	40,9	44,5	48,2	54,5	60,9	57,8	62,9	69,9
Темп роста мирового ВВП	1,0	1,13	1,09	1,09	1,13	1,12	**0,95**	1,09	1,11
Отношение глобальных финансовых активов к ВВП	3,42	3,54	3,42	3,95	4,21	3,52	4,02	3,98	3,66
Индекс изменения отношения финансовые активы/ВВП	1,0	1,04	0,96	1,15	1,07	0,83	1,19	0,99	0,92