XVI April International Academic Conference

NRU-HSE, Moscow

SMIRNOV, Alexander D.[1]

Stochastic Leverage of the Global Financial System

Contemporary world finance according to statistics of the IMF, summarized in *Table1,* is an immensely huge and complex system. The total amount of financial assets in the world has achieved an astronomical figure of $270 trillion in 2012, and almost four times oversized total real resources measured by the world GDP. The scale of the world finance is by orders of magnitude larger than any of its participants including banks and financial corporations with trillions of dollars in their assets. These characteristics reflect the functioning of a highly sophisticated network of banks, companies, markets and instruments encompassing actions of innumerable producers, consumers, investors, creditors and borrowers operated on different financial and real markets.

Probably the most prominent feature of the world financial system development is its continuously growing debt. The world debt, increasing by 5.3 percent annually in 2007-14 (Bloomberg, February 5, 2015), and outrunning growth of the world GDP, has become one of the major economic concerns. By what means is it possible to model the global phenomenon of growing debt? What are the major interrelations among leverage and structure of a financial system? Whether predictions of this enormous and complex process are reproducible and, thus, do they have some scientific value?

Debt process, as a basic relation between creditors and borrowers, is the core of financial intermediation. The latter, in terms of time, is a natural form of manifold processes of investing in a competitive environment, both on financial and real markets. Different financial rates, or

[1] ***SMIRNOV, Alexander D.,*** Doctor of Economic Science, Professor, Member of Russian Academy of Natural Sciences, National Research University – Higher School of Economics, 101000 Moscow, Myasnitskaya street, 24, building 4, e-mail: adsmir@hse.ru

their expected values, are used to describe investing and aggregate saving. In terms of space, financial intermediation is represented generally by leverage[2]. The latter, as a special measure of financial process, has attracted a lot of attention recently. Modeling of debt and financial leverage has become a noticeable feature of modern academic research (Adrian, Shin, 2008; Gromb, Vayanos, 2008; Geanakoplos, 2010; Holmstrom, 2015; Thurner et al, 2011). These works stressed the importance of the causal relationships and feedbacks, as well as the necessity of a general approach towards understanding leverage dynamics and the structure of a financial system.

Table 1. Global Financial System in 2003-12 (US dollars, trillion)

Years	**2003**	**2004**	**2005**	**2006**	**2007**	**2008**	**2009**	**2010**	**2011**	**2012**
Total Assets, A_t	128.3	144.7	151.8	190.4	229.7	214.4	232.2	250.1	255.9	268.6
Index, yoy	1.0	1.17	1.05	1.25	1.21	0.93	1.08	1.08	1.02	1.05
Stocks, e_t	31.2	37.2	37.2	50.8	65.1	33.5	47.2	55.1	47.1	52.5
Index, yoy	1.0	1.19	1.00	1.37	1.28	0.51	1.41	1.17	0.85	1.11
Debts, x_t^1	52.0	57.9	58.9	68.7	79.8	83.5	92.1	94.8	98.4	99.1
Index, yoy	1.0	1.11	1.02	1.17	1.15	1.05	1.10	1.03	1.04	1.01
Bank assets, x_t^2	40.6	49.6	55.7	70.9	84.8	97.4	93.0	100.1	110.4	117.0
Index, yoy	1.0	1.22	1.12	1.27	1.20	1.14	0.95	1.07	1.10	1.06
World GDP, Y_t	36.2	40.9	44.5	48.2	54.5	60.9	57.8	62.9	69.9	72.2
Index, yoy	1.0	1.13	1.09	1.09	1.13	1.12	0.95	1.09	1.11	1.03

{*Source*: IMF, Global Financial Stability Reports, 2003-2014}

The origins of a feedback analysis in economics might be traced back to the logistic equation introduced by *P-F. Verhulst* in the first half of nineteenth century. Since then the logistic model, both continuous and discrete, in its innumerable versions, has been a "workhorse" in biology and other sciences. Yet, paradoxically, the continuous logistic model, with very rare

[2] An excessive leverage was considered, almost unanimously, to be a major culprit of the credit crunch 2007-2008 while interest rates became almost negligible (Cassidy, 2010). Since then vivid discussions have been going on in academic circles as well as in banking community aiming to a systemic monitoring and management of leverage.

exceptions (Aoki, 1998), attracted virtually no attention of economists[3]. Such a negligence was in no way valid, nor justified. Once the continuous stochastic logistic model is defined properly, it is able to encompass parsimoniously important issues of leverage dynamics in the short and long run (Smirnov, 2012, 2013, 2014).

This report was focused on the stochastic logistic model as a unified means to study financial leverage dynamics. Causal relationships and non-linear feedbacks in finance were explained under simpler (zero variance) deterministic assumptions. Basic financial balances of levels and flows were recombined into a logistic model reproducing the market behavior in rational and irrational regimes. The long term random leverage behavior was explained via its asymptotic (stationary) gamma distribution and its characteristics. The model specified various rates of return, their spreads, stationary and anchor leverage, collateral ratios and the natural rate of interest. The case of convergence to the anchor leverage was studied due to its economic importance, especially for effective management and control.

An analysis of the world financial system, as a whole, however disputable, was considered as a feasible alternative between *Scylla* of distortions due to international financial markets and *Charybdis* of regional discrepancies among financial subsystems[4]. In the author's opinion, regional discrepancies were lesser evils and were, at least partially, smoothed by appealing to the world averages.

Basic macrofinancial equations

The starting point of the modeling of a logistic macrofinancial system was formed by the scalar financial balance:

(1) $A(t) = x(t) + e(t)$

that included variables of assets, $A(t)$, debt, $x(t)$, and capital, $e(t)$. Equation (1) has a standard meaning of a balance between value of aggregate assets and total liabilities, including aggregate debt and equity. Evidently, actual debt has some maturity, T, which is an important characteristic of the market solvability and riskiness. Thus, equation (1) was made consistent by assuming an existence of asymptotic relation:

[3] Logistic maps (difference equations) were popular in economic literature in 80ties, especially after publication of a famous paper by R. May on simple equations with a very complicated behavior. Maps, as having very different dynamics in comparison with continuous processes, were not investigated in this paper.

[4] For the same reasons the author did not suggested the Russian banking system as an object for the model verification. Explaining specific features of Russian finance would have driven the reader far away from the focus of the paper.

$\lim_{T \to t} x(t,T) = x(t)$,

and the appropriate equality for assets. Aggregate financial flows, being subject to the equality between saving and investment, were represented by the following equation:

$$dA(t) = dx(t) + de(t)$$

where d is the operator of taking a differential. In terms of future value, being parameterized by instantaneous rates of return[5] on assets, μ, on aggregate debt in money form, r, and on equity, ρ, the balance for financial flows takes a simple form:

(2) $\mu A = rx + \rho e$.

The system (1-2) get a scalar representation by introducing a macrofinancial leverage, $l(t) = A(t)/e(t)$, as a ratio of assets to equities. A linear equation with regard to leverage could be easily arranged:

(3) $\rho = r + (\mu - r)l; \quad 1 \le l < \infty$.

Historically, probably, only the assets of Dutch *Wisselbank* (1603) could have been formed along the borderline case, $l = 1$ (Ferguson, 2009). Modern financial system contains huge amount of debt, $x > 0$, and the macrofinancial leverage in 2003-12 was never smaller than 3.9.

Equation (3) is widely used to boost ROE for a given positive spread $(\mu - r)$. Accordingly, negative spreads that are imminent in periods of critical market turbulence multiply losses of the market participants[6]. Multiplying both sides of (3) by l^{-1} and rearranging terms, parameter μ might be represented as

(3') $\mu = \mu + (\rho - r)l^{-1}$,

which is an arithmetic average of parameters ρ and r, taken with weights l^{-1} and $(1 - l^{-1})$, respectively. Hence parameter μ has a meaning of a weighted average cost of capital, $WACC$. To avoid economic inconsistencies among level and flow variables macrofinancial rates of return are to satisfy either one of the following inequalities:

(4) $\rho > \mu > r$ or $\rho < \mu < r$.

Causality of a logistic model

---

[5] Since it is impossible to fix a "macrofinancial portfolio" , these rates are just analogues of the widespread yields on asset, equity and refinancing rate.

[6] As well known, equation (3) appears in the study of the Modigliani-Miller Theorem. Note, also that, by implying condition, $r = 0$, equation (3) is reduced to $\rho = \mu l$ as in the widespread DuPont model.

For a huge global financial system, operating with virtually millions manifold assets possessing different characteristics, assumptions of continuity and differentiability of a leverage seem to be quite plausible. In accordance with

$\lim_{T\to t} A(t,T) = A(t)$,

the existence of an instantaneous leverage, $l(t)$, can be verified thus leading to a simple relation between leverage and its changes:

$$d\,l(t) = [\mu - \rho]\,l(t)dt\,.$$

This differential equation, together with equation (3), forms the leverage logistic model:

$$(5)\quad dl(t) = [a - b\,l(t)]\,l(t)dt\,;\; l(0) = l_0$$

where $a = (\mu - r)$ is one parameter of the model. The leverage dynamics is governed by the quadratic feedback with the intensity dependent upon the magnitude and sign of parameter b. The feedback parameter $b = (\mu - r)^2 / (\rho - r)$, in order to translate the stationary point to the right from $l = 1$, is smaller than the first spread, $b < a$. Their ratio:

$$(6)\quad l^* \equiv K = \frac{a}{b} = \frac{\rho - r}{\mu - r}$$

defines a stationary point of equation (5) which is a focus of our interest [7]. Since leverage, by economic sense, is a positive number, spreads $\{a, c\}$ are either positive or negative, simultaneously.

For positive spreads, leverage, according to (5), changes non-linearly with drift $a > 0$ and negative feedback parameter $b > 0$. This process reflects difficulties on the normal market for borrowers with large debts in obtaining additional credit. These effects stabilize the debt market forcing leverage to converge to its stationary state (6). If spreads are negative, leverage would change due to positive feedbacks emerging in an irrational market. "Irrational exuberance" is a symptom of the leverage instability. Empirical leverage behavior for years 2007-08 is illustrated in Table 2 composed on the IMF data (GFSR, 2008, 2009).

[7] The other stationary point, $l_1 = 0$, of equation (5) is of no economic importance.

Table 2. Credit Crunch 2007- 08 Parameters

	ρ	μ	r	$a=\mu-r$	$c=\rho-r$	$b=\dfrac{(\mu-r)^2}{\rho-r}$	K
2007	0.2805	0.2064	0.1791	0.0273	0.1024	0.00728	3.75
2008	–0.4854	–0.0666	0.0990	–0.1656	–0.5844	–0.04691	3.53

This data was used for constructing a phase diagram (Figure 1) for the equation (5).

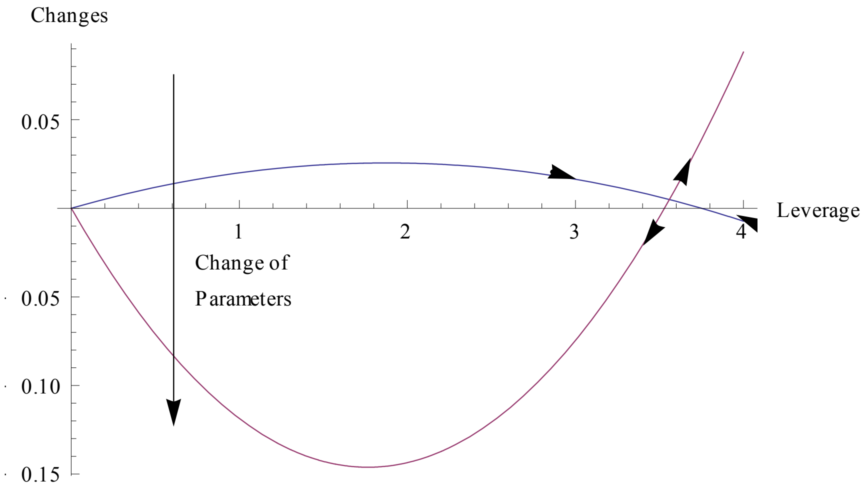

Figure 1. The phase portrait of the global financial system for 2008-09.

It shows that the global leverage reacted to the "overshooting" market effect before the credit crunch and went to instability (crisis) for the year 2008. Thus, the short run leverage dynamics near a stationary point demonstrated the quality of a systemic metastability.

The short run leverage dynamics

In general, the short run dynamics is going on in the vicinity of a stationary leverage. For small changes in leverage, rates and their spreads could be considered constant. By economic meaning, spread $a=(\mu-r)$ is an indicator of credit supply while the spread $c=(\rho-r)$ is an indicator of demand for credit. At a stable stationary leverage l^* for positive spreads indicators of demand for assets (LHS) and the constant supply of credits (RHS) are equal[8]:

(7) $(\rho-r)l^{*-1}=(\mu-r)/$

[8] Evidently, the same results could be achieved for constant indicator of the demand and increasing supply of credits. That makes equation (3') redundant in the analysis of a short run.

Thus, for any $l_1 < l^*$, an investor's position is represented by inequality: $(\rho - r)l_1 > (\mu - r)$, and demand for credit exceeds its supply. Investors would borrow additional funds and their actions increase leverage until it reaches the stationary point. On the other hand, at any $l_2 > l^*$ the demand for loans is less than their supply. Investors, by selling their assets, would drive leverage down to the stationary state. Financial markets with positive spreads behave in accordance with the analysis of the Swedish economist *K. Wicksell* (1898) for a market of credits. Wicksellian differentials, analogous to the spreads in our model, were thoroughly investigated by T. Aubrey (2013).

On the market with negative spread, $a < 0$, stationary leverage is instable and economic logic becomes a bit more complicated since the Jacobian is positive: $\partial f / \partial l = -a > 0$. For any $l_1 < l^*$, we have an inequality: $-(\rho - r)/l_1 < -(\mu - r)$. In terms of losses and economy, which are the positive amounts, the same position should be expressed (after the multiplication by -1) as the opposite inequality: $(\rho - r)/l_1 > (\mu - r)$. Hence if losses exceed economy, a typical investor has to decrease his/her debt exposition by lowering leverage. Such actions would explain investors' behavior along the trajectory of the Fisher debt deflation process (*the Fisher branch*).

Similarly, for any $l_2 > l^*$ negative returns on investors' position is larger than their negative costs: $-(\rho - r)/l_2 > -(\mu - r)$, meaning that the economy on their costs would exceed their losses. Thus investors are stimulated to borrow and increase their leverage but such actions on the abnormal market (with negative spreads) would blow up a financial bubble. The appropriate mechanism of irrational behavior was described by *H. Minsky* (1986). Situations, described above, are represented on Figure 2 based upon empirical data of Table 2.

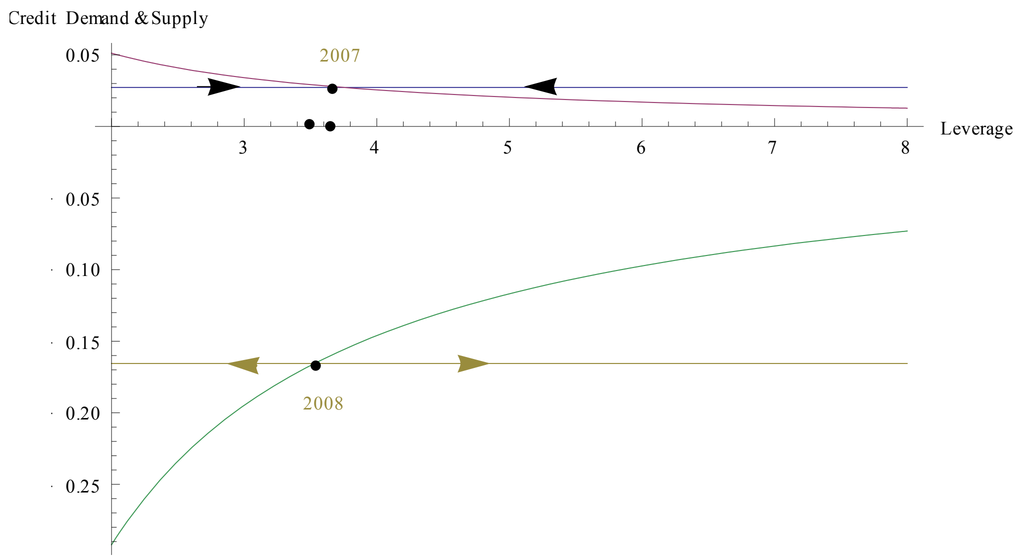

Figure 2. Attractor and repeller for the global finance in 2007-08.

As known, logistic equation (5), though nonlinear, has an analytic solution[9]. The family of logistic trajectories is represented graphically on Figure 3. Each trajectory $l(t)$ is specified by its initial state, l_0, and spread, a, according to

$$(8)\quad l(t) = l^*\{1 + (\frac{l^*}{l_0} - 1)\exp[-(\mu - r)t]\}^{-1}.$$

Solution (8) was written as a weighted harmonic average of the initial leverage, l_0, and its stationary state, l^*. Note that the bubble (or the Minsky) branch exists for negative parameters and $l_0 > l^*$, the debt-deflation (or the Fisher) branch exists for negative parameters and $l_0 < l^*$, while stable trajectories exist for positive parameters $\{a, b\}$.

All in all, as shown in Figure 3, the logistic model helps to classify different market regimes consistently. For the normal (*Wicksellian*) market spreads are positive and the stationary leverage is stable. Hence investors would borrow additional funds (thus increasing their leverage) if the demand for credit exceeded its supply, and vice versa. Stationary leverage is instable on the abnormal market[10] where growing prices lead to increasing demand for assets (along the Minsky bubble) or to the Fisherian debt deflation trajectory (Fisher, 1933). Spreads

[9] The logistic (or Verhulst) equation is a particular case of a Riccati equation.

[10] Vertical asymptote in Figure 3 exists only for negative parameters and $l_0 > l^*$.

are negative on the irrational market. Thus, contrary to the "Levins paradox"[11] the financial leverage model did allow for the existence of negative parameters (spreads) that would give rise to bubbles and ultimate market collapse. These situations are abnormal, short lived, and highly undesirable but, nonetheless, they could have arose aperiodically on financial markets.

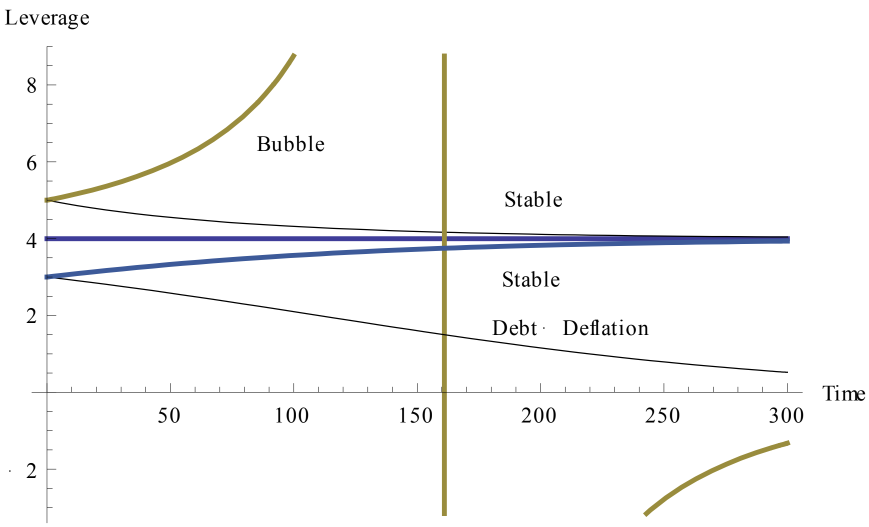

Figure 3. Logistic trajectories of leverage.

Stochastic leverage and its long run dynamics

Under uncertainty market participants are unable, for a however short period of forecasting, to predict future leverage without some error. A continuous random process of leverage, $L(t)$, follows the stochastic differential equation, SDE:

(9) $dL(t) = L(t)\{[a - bL(t)]dt + \sigma dW(t)\}$,

where σ is the leverage volatility, and $W(t) = \int_0^t dW_u$ is the standard Brownian motion[12].

Loosely speaking, SDE (9) says that change in the leverage, $dL(t)$, is composed of a non-linear, non-random drift, $[aL(t) - bL^2(t)]dt$, and a diffusion, $\sigma L(t)dW(t)$, influenced by a differential, $dW(t)$, of Brownian motion.

---

[11] Populations cannot grow without resources available; thus negative parameters were excluded from the analysis of logistic models in biology (Gabriel et al, 2003).

[12] Since Brownian motion has no derivative equation (9), in fact, is a symbolic representation of the integral equation: $L(t) - L(0) = \int_0^t L(u)[a - bL(u)]du + \int_0^t \sigma L(u)dW(u)$.

Stochastic processes, defined by SDE (9), have an asymptotic (or stationary) distribution. Contrary to a single instrument, the global financial system exists, by definition, “almost forever”. Hence, the study of a stationary distribution of aggregate leverage reveals important features of its long run behavior. Stationary probability density function, $p(l)$, of a random leverage, $L(t)$, could be found by solving the forward Kolmogorov equation (FKE):

$$(10)\quad -\frac{\partial}{\partial l}[l(a-bl)p(l)]+\frac{1}{2}\frac{\partial^2}{\partial l^2}[\sigma^2 l^2 p(l)]=0 .$$

Contrary to the general FKE, equation (10) is an ordinary differential equation[13], and has two solutions (Pascuali, 2001). The trivial solution, $p(l)=\delta(l)$, is a $\delta-Dirac$ distribution associated with the stationary state, $l_1^*=0$. A non-trivial solution to equation (10) happens to be the *pdf* of a gamma-distributed leverage:

$$(11)\quad p(l)=[\beta^{\alpha}\Gamma^{-1}(\alpha)]l^{\alpha-1}e^{-\beta l}$$

where α, β are parameters of a distribution, and $\Gamma(.)$ is gamma function. Gamma distribution is defined for positive parameters of shape, α, and rate, β (or scale $1/\beta$), respectively: $\alpha=\frac{2a}{\sigma^2}-1;\ \beta=\frac{2b}{\sigma^2}$. Hence gamma distribution exists on an interval: $0<\sigma^2<2a$, in several modifications. Contrary to the normal distribution, the most probable asymptotic characteristic of gamma distributed random leverage is its mode:

$$(12)\quad Mode[L]=\frac{\alpha-1}{\beta}=K-\frac{\sigma^2}{b};\alpha>1$$

which is smaller than its expectation:

$$(13)\quad \langle L\rangle=\int_0^{\infty} l\, p(l)dl=\frac{\alpha}{\beta}=K-\frac{\sigma^2}{2b}.$$

The mode exists for “peaked” modifications of gamma distribution, and is invariably zero, if $\sigma^2=a$. It implies that the mode is an attractor of a convergent random logistic leverage.

In the following the leverage convergence will be analyzed only for positive spreads, $a>0$. This simplification is not too restrictive since financial crises, defined by negative spreads, are

[13] Physicists usually call forward Kolmogorov equation (FKE) a Fokker-Planck equation.

short-lived economic phenomena. As a consequence, the "babble" trajectory in Figure 5 exists only in the short run.

The Lyapunov exponent and the leverage convergence

For a gamma distributed leverage its Lyapunov exponent:

$$\langle \lambda \rangle = \frac{\beta^{\alpha}}{\Gamma(\alpha)} \int_0^{\infty} (a - 2bl) l^{\alpha-1} \exp[-\beta l] dl$$

can be easily reduced to a simple difference:

(14) $$\langle \lambda \rangle = \langle a - 2bL \rangle = a - 2b\langle L \rangle = a - 2b(\frac{a-\sigma^2}{b}) = \sigma^2 - a \, .$$

In plain words, if expected variance of leverage is less than its spread, $a > 0$, then a stochastic leverage would converge to its mode in the long run (Dennis et al, 2003). The Lyapunov exponent of a logistic leverage model serves as a measure of investors' confidence in the long term market solvency, the latter being associated with the ultimate debt redemption. Investors are confident in the market solvability if the Lyapunov exponent is negative. But, the latter becomes positive if "the market stay irrational longer than you remain solvent".

Anchor leverage and the natural rate of interest

Financial stabilization in the model is a process contingent on the leverage convergence to the mode of its stationary gamma distribution. How could this mathematical result be explained from the economic point of view?

Consider a macrofinancial system for $\rho > \mu > r$ that represented by some configuration $\{r, \mu, \rho; l^*\}$ with empirically given constant rates of return and leverage, l. In the short run, both spreads $\{a, c\}$ are constant, and the leverage can take small changes near its stationary state. Thus, one of the equations, (3) or (3'), is redundant. Evidently, it is not plausible in the long run for creditors and borrowers being motivated by only one of the two spreads. Creditors behavior, in aggregate, is influenced by spread, $a = (\mu - r)$, and would define the rate of return on equity, $\rho \equiv ROE$. The latter, similar to microlevel, could increase via accumulation of aggregate debt, hence, the leverage increase. Large changes in leverage in combination with constant spread a could change the return on equity, $\rho(l)$, substantially:

(15) $$\rho(l) = r + (\mu - r)l \, ; \rho(1) = \mu \, .$$

On the other hand, the same initial configuration, $\{r, \mu, \rho; l^*\}$, defines the collective behavior of borrowers, the latter being subject to another constant spread, $c = (\rho - r)$. In aggregate, borrowers would influence the rate of return on assets, $\mu \equiv ROA$:

(16) $\mu(l^{-1}) = r + (\rho - r)l^{-1}; \mu(1) = \rho$.

From the long run perspective, every actual market configuration $\{r, \mu, \rho; l^*\}$ could be seen as containing in itself a germ of a continuum of consistent returns on assets and equities that would evolved in a process of mutual adjustments of creditors and borrowers. Evidently, equations (15, 16) have a common positive root, l_N, that could be found by solving an equality:

(17) $\nu \equiv \mu(l_N^{-1}) = \rho(l_N)$.

In words, returns on equity, ROE, and on asset, ROA, are equal at *the anchor leverage:*

(18) $l_N = \left(\dfrac{a}{b}\right)^{0.5} \equiv K^{0.5} = \left(\dfrac{\rho - r}{\mu - r}\right)^{0.5}$.

Their common value, ν, could be called *the natural rate of interest* from the following reasons. In the region of overindebtedness inequality $ROA < \nu$ is taken place. If return on assets is associated with the current interest rate, then this inequality would correspond to the credit expansion triggering inflation, ultimately. The opposite inequality, $ROA > \nu$, takes place for overcapitalized systems and would define excess saving triggering deflation. The above said has clear allusions to the Wicksellian analysis of money expansion, inflation and deflation.

Convergence to the anchor leverage

Convergence towards anchor leverage is tantamount to the financial system stabilization in the long run. In its turn, anchor leverage is an attractor for a stochastic logistic dynamics subject to constraints upon its variance. If a random logistic process converges towards anchor leverage:

(19) $Mode[L] = l_N$,

then the critical variance should be equal to

(20) $\sigma_c^2 = a - \sqrt{ab}$.

At the critical variance, $\sigma_t^2 = \sigma_c^2$, financial market deteriorates with gamma distribution being reduced to the exponential one. Some results of numerical simulation of this particular case of a long run convergence are presented in Table 3.

Table 3. Results of simulation on actual data.

a	b	$K = a/b$	$\sqrt{K} = l_N$	σ_c^2	α	β	$1/\beta$	$\langle \lambda \rangle$	$\langle L \rangle$	$Mode$	ν
0.034	0.0063	5.4	2.34	0.019	2.58	0.663	1.51	–0.015	3.89	2.34	0.0691

Empirical value of the anchor leverage $l_N = \sqrt{K_H}$ appeared to be quite close to 2.0 that means the highest backup of a debt by the total capital. As seen from Table 4, the anchor leverage is associated with elementary probability of 20 percent per annum which is the most robust global financial market configuration under this scenario.

Table 4. Probabilities of different market configurations

	l_1	l_2	l_3	$l_N = Mode$	$\langle L \rangle$	K
Leverage	1.0	1.5	2.0	2.34	3.89	5.4
Probability	0.13	0.17	0.197	0.2	0.16	0.01

Major results of the model simulation were represented in Figure 4. In particular, given empirical values of the global market parameters, it defined the most probable state of a global leverage, its expected value and the deterministic attractor.

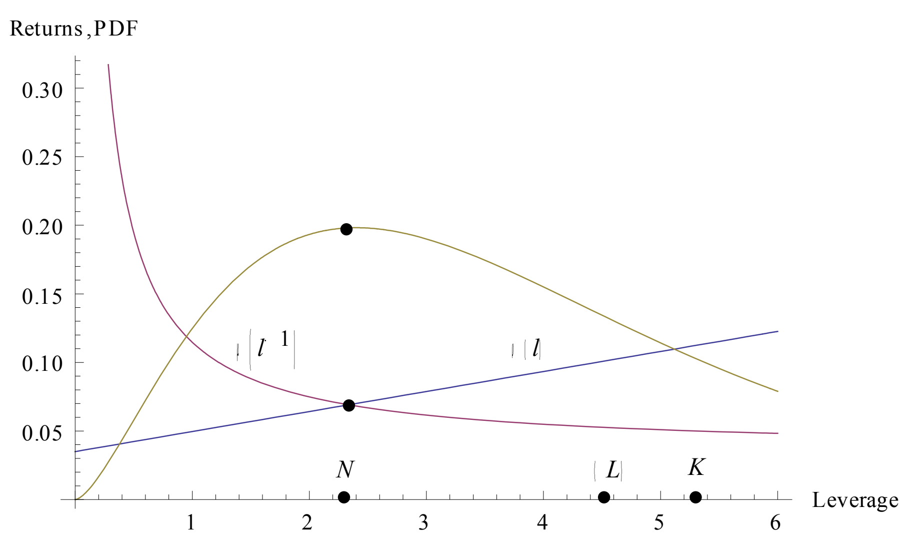

Figure 4. Anchor leverage as an attractor of a gamma distributed model.

Collateral ratio for global finance

The data of Table 1 was used for estimating empirical quantities like financial leverage, l_t, coefficient q_t, and collateral ratios, l_Y. These quantities were reproduced in Table 5.

Table 5. The Global Collateral Ratio components

Годы	**2003**	**2004**	**2005**	**2006**	**2007**	**2008**	**2009**	**2010**	**2011**	**2012**
l_Y	3.42	3.54	3.42	3.95	4.21	3.52	4.02	3.98	3.66	3.72
l_t	3.97	3.89	4.08	3.74	3.53	6.4	4.92	4.54	5.43	5.12
q_t	0.87	0.91	0.84	1.05	1.19	0.55	0.82	0.88	0.67	0.73
$l_Y = l_t * q_t$	3.45	3.54	3.42	3.93	4.2	3.52	4.03	3.99	3.64	3.74

Aggregate collateral ratio is a simple measure of correspondence between values of total financial assets and real resources, the latter approximated by the world GDP. When empirical ratio is less than its critical value, an economic development, seems to be robust. On the other hand, for an empirical ratio being larger than its critical value, many financial assets become toxic triggering sequence of events finally culminated in a financial crisis. In Table 5 macrofinancial collateral ratio was defined by the following simple expression:

(21) $l_Y = l_t * q_t$,

where $q_t = e_t / Y_t$ is the ratio of total equities to the world GDP. The model of the q dynamics on the microlevel was described in detail by J. Tobin (1996) who suggested an explanation of the equities market fluctuations. Firms financing their activities on equities markets, would expand their investments, if $q_t > 1$. On the other hand, growing cost of investment would drive this coefficient down. If the decrease is significant, and $q_t < 1$, the opposite forces would dominate and drive up this coefficient. Stochastic fluctuations on the equities market seems to be supported by the data assembled in Table 5.

In a formal language it means that coefficient $Q(t)$ would follow the Ornstein-Ulenbeck process with SDE:

(22) $dQ(t) = \kappa[1.0 - Q(t)]dt + \sqrt{D}dW(t)$

where κ is the mean reverting parameter, D is the diffusion parameter, and 1.0 is the long run attractor. Solution to (22) is the following:

$$Q(t) = 1 + (Q_0 - 1)\exp[-\kappa t] + \sqrt{D}\int_0^t \exp[\kappa(u-t)du$$

which evidently has 1.0 as its long term expectation.

(23) $\lim_{t\to\infty}\langle Q(t)\rangle = 1.0$

The long term expectation (23) has a profound effect upon the global collateral ratio (21). Together with the global leverage the latter would converge to the anchor leverage:

(24) $\lim_{t\to\infty} l_Y(t) = l^* \equiv K^{0.5}$.

The collateral ratio of the global financial system converged had an empirical attractor, or the anchor leverage, $K^{0.5} \cong 2.32$. The model suggests that a financial leverage, precisely, determined the basic configuration of the world economic development.

The same reasoning was used to estimate the amount of the world "toxic" assets, l_Y**,** for the year 2012**.** The leverage expectation, $\langle L\rangle = 4.67$, being multiplied by the coefficient, $q_{2012} = 0.73$, gave the "idealized" collateral ratio, $l_Y = 3.41$. The latter, in fact, corresponded to the actual world GDP in \$72.2 tn. It means that in the year 2012, the world assets were fully collateralized in amount of $A = \$246.2\,tn$. In other words, the world financial system contained about \$12.4 tn (or approximately 5 per cent) of toxic assets.

Some conclusions

A sample about the world financial system, assembled in Table 1, was small but had a well defined structure that helped to construct the stochastic logistic model. The latter, being established upon mild assumptions about causation, feedbacks and structure of macrofinancial processes, was able to encompass parsimoniously the morphology (Goldsmith, 1969) of a world financial system. It produced, via logistic SDE for leverage and FKE for its density probability function, a subordinated sequence of characteristics including deterministic and stochastic leverage attractors, expected leverage, global collateral ratios and the natural rate of return. By assigning asymptotic characteristics to a large and complex world financial system, the model provided invariants useful for efficient implementation of a consistent policy of financial stabilization.

The model explained persistent accumulation of the world debt and discovered no presence of so called "debt cycles". Quite possibly, the small size of a sample data has to be blamed for this "disappointing" result. Yet, a prominent deviance from the anchor leverage was in agreement with annual empirical variance ($\sigma^2 = 0.081$) and the spread, $a_H = (\mu - r) = 0.034$. Hence the global drift to ever larger aggregate debt has to be taken seriously, unless forcible factors of the leverage convergence will be found and identified.

References

Adrian, T., Shin, H., S. (2008) Liquidity and financial contagion, Banque de France, *Financial Stability Review*, # 11, February, Paris

Aoki, M., (1998) New Approaches to Macroeconomic Modeling, Cambridge University Press, Cambridge

Aubrey, T. (2013) *Profiting from Monetary Policy: Investing through the Business Cycle*, Palgrave Macmillan, London

Cassidy, J. (2009) *How Markets Fail. The Logic of Economic Calamities*, Allen Lane, London

Dennis, B., Desharnais, R., Cushing, J., Henson, S., Constantino, R. (2003) Can Noise Induce Chaos? *OIKOS* 102:2, p. 329-339

Ferguson, N (2009) *The Ascent of Money*. A Financial History of the World, Penguin Books, London

Fisher, I. (1933) The Debt-Deflation Theory of Great Depressions, *Econometrica*, 1, pp.337-57.

Gabriel, J-P., Sausy, F., Bersier, L-F. (2003) *Paradoxes in the Logistic Equation?* Comment, doc.rero.ch/record/4133/files/1_ gabriel_plc.pdf

Geanakoplos, J. (2010) Solving the Present Crisis and Managing the Leverage Cycle, *FRBNY Economic Policy Review,* August, New York

Global Financial Stability Report, issues 2004-2013, IMF, Washington, D.C.

Goldsmith, R.W. (1969) Financial Structure and Development, Yale University Press, New Haven

Gromb, D. and Vayanos, D. (2008) Leverage and Liquidity Dry-Ups: A Framework and Policy Implications

Holmstrom, B. (2015) Understanding of the Role of Debt in the Financial System, BIS Working Papers, #479, www.bis.org

Minsky, H. (2008) *Stabilizing an Unstable Economy*, 2nd edition, McGraw Hill, New York.

Pasquali, S. (2001) The Stochastic Logistic Equation: Stationary Solutions and their Stability, *Rendisconti del Seminario Matematico della Universita di Padova*, vol.106, pp. 165-183

Smirnov, A.D. (2012) Leverage and Financial Instability, *Voprosy Economiki*, #9, 25-40, Moscow, in Russian

Smirnov, A.D. (2013) Logistic Model of Financial Leverage, *HSE Economic Journal,vol.17, #4,* 585-616, Moscow, in Russian

Smirnov, A.D. (2014) Financial Assets Collateralization and Stochastic Leverage, *HSE Economic Journal,vol.18, #2,* 183-215, Moscow, in Russian

Thurner, S., Farmer, D., Geanakoplos, J. (2012) Leverage causes fat tails and clustered volatility, *Quantitative Finance*, vol. 12, no. 5, May 2012,695-707

Tobin, J. and Brainard, W. (1977) Asset markets and the cost of capital, in *"Economic Progress, Private Values and Public Policy",* North-Holland, Amsterdam

K. Wicksell (1898) *Interest and Prices* (http://mises.org/books/interestprices.pdf), Ludvig von Mises Institute, 2007