

# FORMAL SOLUTIONS TO INTEGRABLE HIERARCHIES

SERGEY M. NATANZON

Formal solutions we understand as a formal Taylor series that satisfies all equations of hierarchy. This class contain, of course, all analytical solutions. But this class contains also some interesting series, which are non-convergent. This, in particular, generating functions for Hurwitz numbers, generating functions for intersection indexes on moduli space of curves and other.

In the first lection I will tell, that formal solutions of KP hierarchy are defined by denumerable set of functions from one variable. Moreover, formal solutions of n-KP hierarchy are defined by n-1 functions from one variable. This, after Witten-Kontsevich theorem, gives possible to find intersection indexes on moduli space of complex curves with marked points.

In the second lection I tell, about formal solutions of  $\hbar$ -KP hierarchy. The  $\hbar$ -KP hierarchy is a set of integrable hierarchies continuously depending from auxiliary parameter  $0 \leq \hbar \leq 1$ . For  $\hbar = 1$  this is KP hierarchy, for  $\hbar = 0$  this is dispersionless KP hierarchy. For any  $\hbar$ , formal solutions of  $\hbar$ -KP hierarchy also are defined by denumerable set of functions from one variable. Moreover, A.Zabrodin and I found formula for this solution.

In the last lection I tell, about symmetric solutions of dispersionless 2D Toda hierarchy. The symmetric solutions are a natural class of solutions that contain, in particular, a string solution and (according Okounkov) a generating function for 2-Hurwitz numbers of genus 0. A.Zabrodin and I proved, that symmetric solutions are defined by one function from one variable. We found also a representation of the solution by this function. As application, this gives a formula for the 2-Hurwitz numbers. The other application is a formula for conformal map of any simply connected complex domain with smooth boundary to standard complex disk.