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Stochastic Leverage Model: Applications to the Global Financial System Dynamics

Moscow, November 15, 2016

Leverage and Financial Instability
Voprosy Economiki, #9, 1-27, 2012 (in Russian)

Logistic Model of Financial Leverage
HSE Economic Journal,vol.17, #4,585-616, 2013 (in Russian)

Financial Assets Collateralization and Stochastic Leverage
HSE Economic Journal,vol.18, #2, 183-215, 2014 (in Russian)

Stochastic Leverage of the Global Financial System,
Proceedings of XVI International April Conference, NRU HSE, 732-741, Moscow, 2016

Stochastic Logistic Model of the Global Financial Leverage,
submitted to *The BE Journal of Theoretical Economics*, January 14, 2016

Entia non sunt multiplicanda
praeter necessitatem
Lex parsimoniae

Herewith is presented a model of macrofinancial leverage dynamics in the short and the long run

The model facilitates analysis of the Global Financial System, including consistent estimates of its failure and survival, collateral ratio and anchor leverage.

Database used: ***The IMF***, *Global Financial Stability Report*, statistical appendices 2003-13.

The research is aimed to make leverage modeling an important part of financial monitoring and control.
These subjects are lively discussed by the **BIS** and the **Fed.**

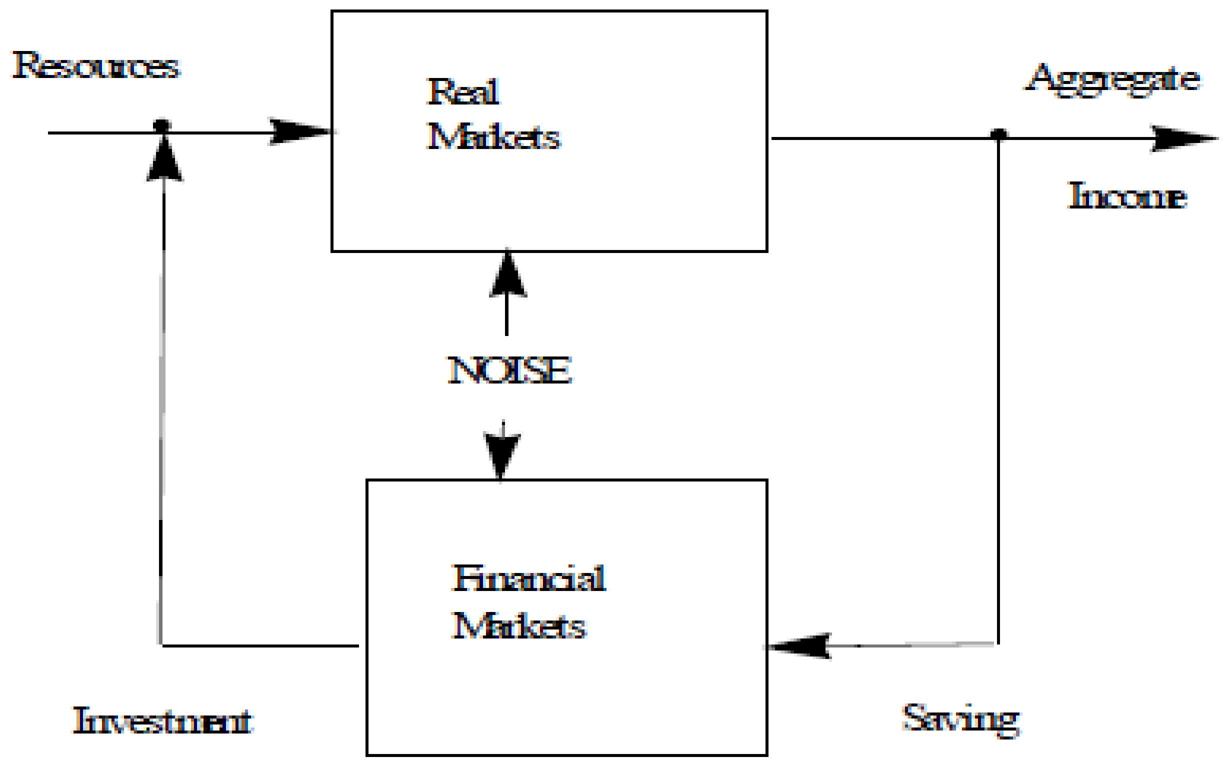

Resources
Real
Markets
Aggregate
Income
NOISE
Financial
Markets
Investment
Saving

Table 1. Global Financial System in 2003-13 (US dollars, trillion)

Years	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013
Total Assets, A_t	128.3	144.7	151.8	190.4	229.7	214.4	232.2	250.1	255.9	268.6	282.8
Stocks, e_t	31.2	37.2	37.2	50.8	65.1	33.5	47.2	55.1	47.1	52.5	62.6
Debts, x_t^1	52.0	57.9	58.9	68.7	79.8	83.5	92.1	94.8	98.4	99.1	99.8
Bank assets, x_t^2	40.6	49.6	55.7	70.9	84.8	97.4	93.0	100.1	110.4	117.0	120.4
World GDP, Y_t	36.2	40.9	44.5	48.2	54.5	60.9	57.8	62.9	69.9	72.2	74.6

{*Source*: IMF, *Global Financial Stability Report*, annual issues 2004-2014}

The logistic model generalizes ***the Malthusian*** hypothesis of economic development subject to ***the Verchulst*** feedback loops.

General approach to the logistic model of leverage dynamics

a) The leverage differential, $dl(t)$, and its level, $l(t)$ are connected in time via $dl = f[l(t)]dt$;

b) RHS is decomposed to represent *the coupling* (in terms of the complex systems theory) between the leverage and its rate of change: $f[l(t)] = l(t) * g[l(t)]$;

c) A Taylor series expansion of $g[(l(t)]$ at $l(t) = l^*$, where $g(l^*) = 0$:

$$g(l) \cong g(l^*) + g'(l^*)(l - l^*) = -g'(l^*)l^* + g'(l^*)l = a - bl$$

leads to the **logistic model of leverage dynamics** in the short run:

$$dl_t = l_t(a - bl_t)dt,$$

where $a = -g'(l^*)l^*$ и $b = -g'(l^*)$.

Russian (Soviet) experiments in 20-ties (XX century) with logistic curves of economic rate of growth

Table 2. Spreads, leverage, its change and rate of change in 2003-2013

Years	Leverage, l_t	Rate, $\Delta l_t / l_t$	Capital Intensity, w_t	Rate, $\Delta w_t / w_t$	Spread, $a = \mu - r$	Spread, $c = \rho - r$	Parameter, $b = a^2 / c$
2003	3.97		0.252				
2004	3.89	-0.020	0.257	0.02	0.0207	0.0852	0.005
2005	4.08	0.049	0.245	–0.047	-0.0169	-0.066	-0.0043
2006	3.74	-0.083	0.267	0.089	0.0363	0.1474	0.0088
2007	3.53	-0.056	0.283	0.059	0.0273	0.1024	0.0072
2008	6.4	0.813	0.156	–0.449	-0.1656	-0.5844	-0.0468
2009	4.92	-0.231	0.203	0.301	0.0598	0.3858	0.0093
2010	4.54	-0.077	0.220	0.084	0.0242	0.1145	0.0051
2011	5.43	0.196	0.184	–0.164	-0.0481	-0.2165	-0.0107
2012	5.12	-0.057	0.195	0.059	0.0146	0.0797	0.0027
2013	4.52	-0.117	0.221	0.133	0.033	0.1734	0.0063

Fig 1 shows linear regression:

$$\Delta l_t / l_t = 0.877 - 0.183 l, \qquad (2)$$

with parameters estimated on given data. Assumption of negative feedbacks in the debt market made the sign of parameter b positive, and the logistic model (2) was checked in a one-sided test of null hypothesis, $H_0 : b = 0$, against its alternative $H_1 : b > 0$. The test results are given in Table 3.

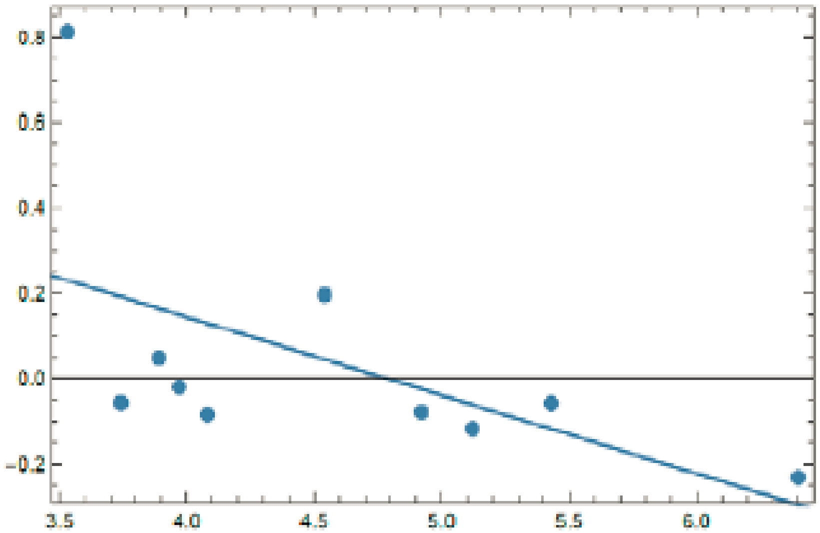


Fig. 1 Linear regression of leverage and its rate of change.

Table 3. Characteristics of the linear model fit

	estimate	standard error	t statistic	p value
1	0.877112	0.438707	1.99931	0.080602
b	-0.183159	0.0945117	-1.93795	0.0886262

General logistic model underscores the importance of feedbacks but fails to provide a convincing description of a financial market.

Raymond Goldsmith , an American scholar, approximately half a century ago proposed the study of *financial morphology* as an instrument of understanding factors of economic growth and development.

Basic equations of **macrofinancial leverage model** consists of the balance of states

$A(t) = x(t) + e(t)$

where $A(t)$ is assets, $x(t)$ is debt and $e(t)$ is capital,.

The aggregate balance of financial flows:

$dA(t) = dx(t) + de(t)$

has an interpretation as

Aggregate Saving = Aggregate Debt + Aggregate Investment

Basic financial rates of return:

$ROE \equiv \rho;\ and\ \ de(t) = \rho\, e(t)dt;$,

$ROA \equiv \mu;\ and\ \ dA(t) = \mu A(t)dt$,

$ROI \equiv r$; and $dx(t) = r\,x(t)dt$

and leverage $l(t) = A(t)\,/\,e(t)$.

form *the balanced financial market* equation

$\rho = r + (\mu - r)l$.

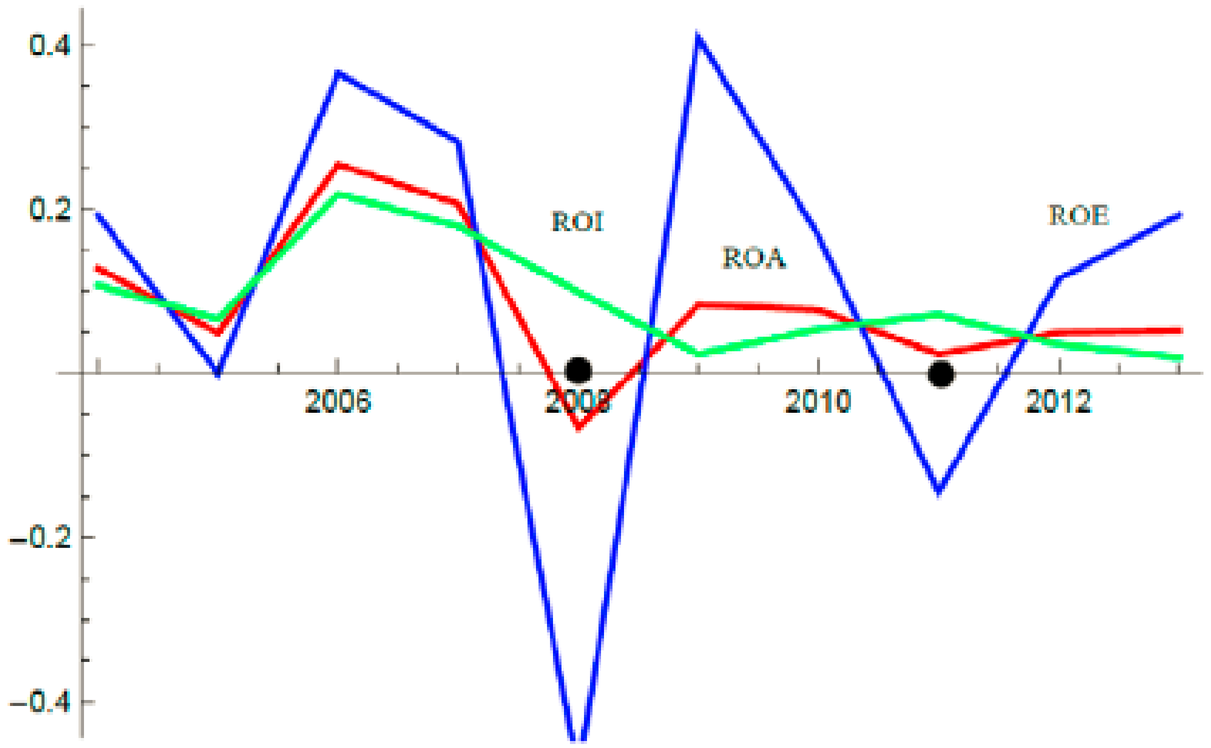


Fig. 2 Global rates of return (computed on the *GFSR* data)

The leverage dynamics in *the short run* is governed by the following differential equation:

$$dl(t) = (\mu - \rho) l(t) dt,$$

which being added with a *feedback loop*:

$$\mu - \rho = (\mu - r)[1 - l]$$

forms **the nonlinear model** with stationary leverage l^* and initial state l_0:

$$dl(t) = (\mu - r)[1 - \frac{l(t)}{l^*}] l(t) dt; \; l(0) = l_0.$$

The alternative representation of a leverage model is

$$dl(t) = a[1 - \frac{1}{K} l(t)] l(t) dt \equiv [a - b\, l(t)] l(t) dt$$

where $l^* \equiv K = a/b = \frac{\rho - r}{\mu - r}$ is the stationary leverage;

$a = \mu - r$ is the WACC/refinancing (deposit) spread;

$aK = \rho - r$ is the ROE/refinancing (deposit) spread;

$b = \frac{(\mu - r)^2}{\rho - r}$ is the leverage drag parameter.

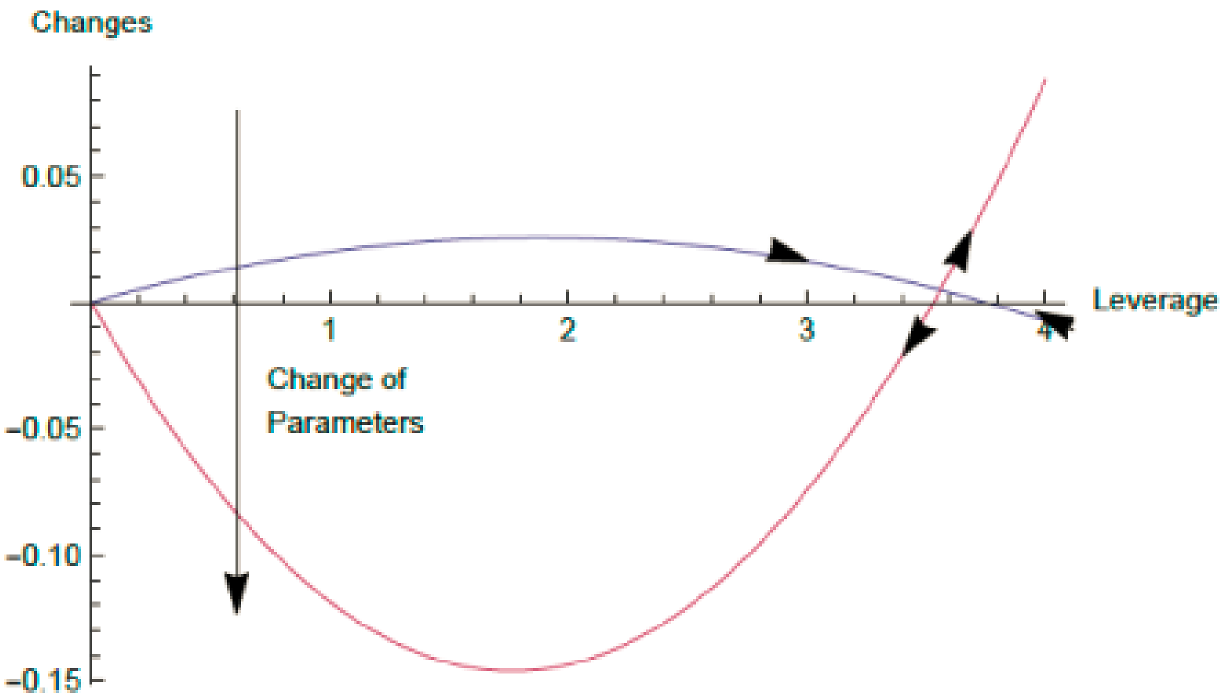


Fig. 3 Phase portrait of the global financial system in 2008-09

Mergers and acquisitions (M&A) market contains equivalent strategies to purchase one of the two companies, with or without debt, L or U, respectively. The companies have the same assets $A_U = A_L = A$ and generate the same return ΔA.

Strategies are represented in the following table.

	t	$T = t+1$
Buy company L	$-e_L \equiv -e$	$+\rho e_L = \rho e$
Buy company U	$-e_U = -A$	$+\rho_U e_U = \mu A$
Borrow at $r > 0$	$+x$	$-r x$
	$-e = -A + x$ $e = A - x$	$\rho e = \mu A - r x$ $\rho = r + (\mu - r) l$

As demonstrated the strategies give rise to ***a standard leverage equation***:

$\rho = r + (\mu - r) l$.

In particular, if buyer's loan equals to debts of company L, $x = x_L$, then the ***Modigliani – Miller, MM, Proposition*** takes place.

It is easy to notice that the widespread **DuPont model**:

$$\rho = \frac{net\ income}{capital} = \frac{net\ income}{sales} \times \frac{sales}{assets} \times \frac{assets}{capital} \quad \text{or} \quad \rho = \frac{\Delta A}{A} \times \frac{A}{e} = \mu l\,,$$

is a particular case of a standard leverage equation, for $r = 0$.

The popular rule-of-thumb: *"A doubling of the capital a bank has will, all else being equal, halve the bank's return on equity (ROE)"*
{*The Economist*, Sep 28th, 2013}:

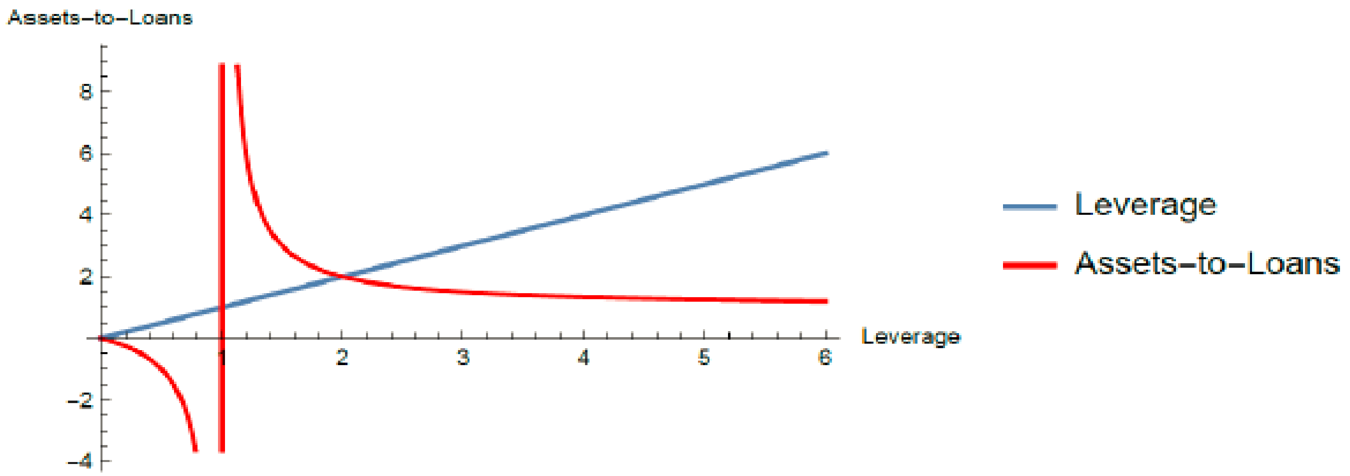
$$\frac{\rho}{2} = \mu \frac{A}{2e}\,.$$

Collateralized loan with margin calls or a marginable loan:

$H = L + M$, where $\lambda = H / M$; $\lambda - 1 = L / M$,

and $H / L = H / M : M / L = \dfrac{\lambda}{\lambda - 1}$.

In the ***US stock market*** brokers/dealers are not allowed to borrow more than 100% of its own capital, hence have their leverage $\lambda \leq 2$. This regulation is explained by equation: $\lambda = \lambda / (\lambda - 1)$



W. Shakespeare, The Merchant of Venice

It is useful to compare the model parameters with rates like yield-to-maturity (YTM), γ, current yield, δ, and investment return, r. As known, in terms of future values, the linear debt model:

$$d\,x(t) = (\gamma - \delta)x(t)dt \quad (7)$$

claims the equality of the rate return on investment, *ROI*, to the difference between *YTM* and current yield, or $d\,x / x \equiv r \equiv \gamma - \delta$. Since model (7) refers to structure of the debt only while equation (6) establishes general relation among assets, capital and liabilities, debt including, the rate of return on investment, $r \equiv ROI$, is subject to the following condition:

$$r = \gamma - \delta = (l-1)^{-1}[\mu l - \rho].$$

The aggregate behaviour of creditors and borrowers, as illustrated in Fig. 4 drawn on the empirical data, could be explained in standard economic terms. The short run dynamics of collective investors' behaviour near stationary leverage is explained via indicators of debt supply, $(\mu - r)$, and demand for debt, $(\rho - r)/l$. Their macrofinancial balance at stationary leverage (9) takes the following form:

$$(\rho - r)/l^{*} = (\mu - r) \qquad (11).$$

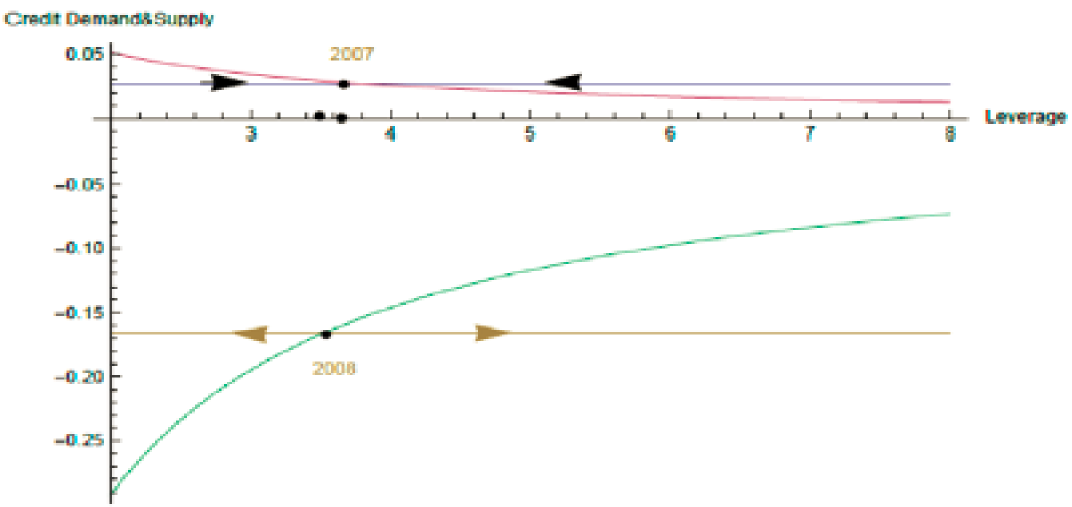


Fig. 4 Attractor and repeller of the global finance in 2007-08

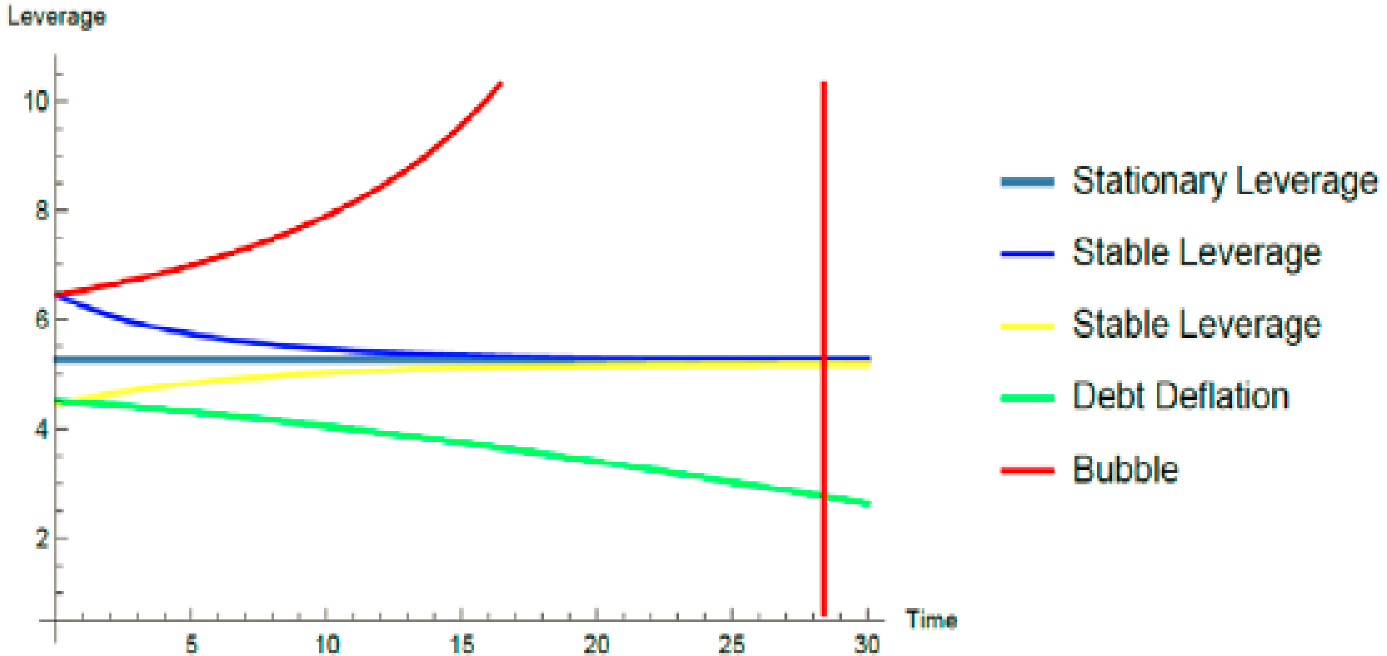
Overall, different debt market regimes in the long run are represented, consistently and comprehensively, by the family of logistic trajectories:

$$l(t) = K\{1 + (\frac{K}{l_0} - 1)\exp[-at]\}^{-1} \qquad (12)$$

where each trajectory $l(t)$ is specified by its initial state, l_0, and spread, a. Alternatively, any solution (12) can be written as a weighted harmonic average[4] of initial leverage, l_0, and stationary state, K:

$$l(t) = l_0 K\{l_0(1 - \exp[-at]) + K\exp[-at]\}^{-1} \qquad (12').$$

The family of trajectories (12) with empirical parameters: $l^* \equiv K = 5.27$; $l_0^1 = 6.45$; $l_0^2 = 4.52$; $a_1 \equiv a_H = 0.0598$; $a_2 = -0.1656$ is represented in Fig. 5 . Different market regimes within the


Leverage
10
8
6
4
2
5
10
15
20
25
30
Time
Stationary Leverage
Stable Leverage
Stable Leverage
Debt Deflation
Bubble

Leverage dynamics is usually "contaminated" with a ***multiplicative noise*** as in:

$$dL(t) = L(t)\{[a - bL(t)]dt + \sigma\, dW(t)\},$$

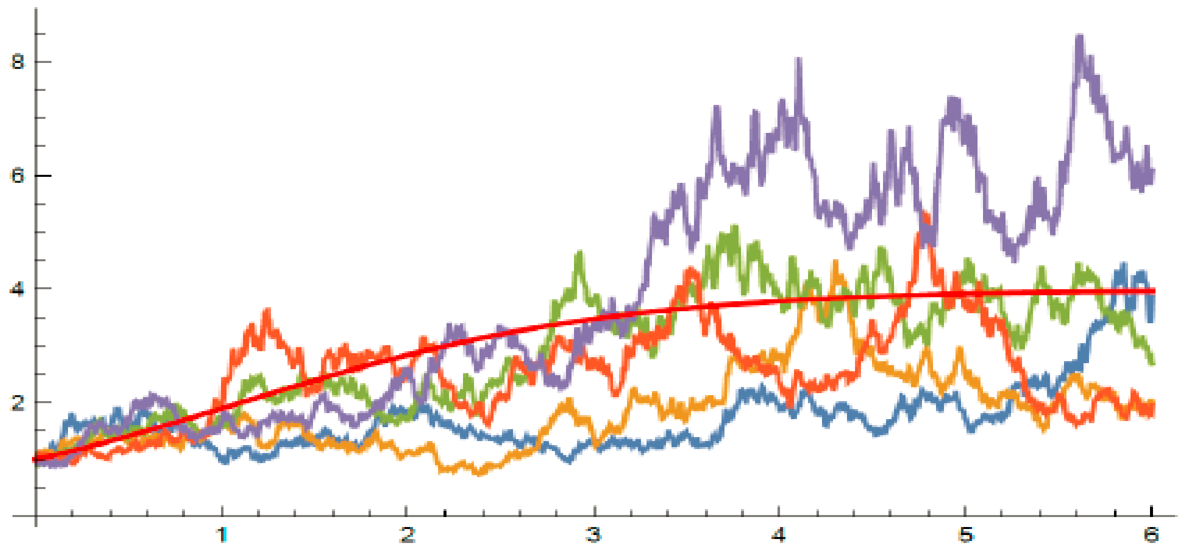
where σ is ***the volatility of leverage***; $zW(t) = \int_0^t dW_u$ is the standard Brownian motion: $dW(t) = \varepsilon_t (dt)^{0.5}$, $\varepsilon_t \sim niid(0, \sigma^2)$.

Since $dW^2(t) = dt$, the standard Brownian motion is a nonstationary process.

The "strong" solution of the stochastic logistic equation (Sciadas, 2010):

$$l(t) = \frac{l_0 \, K \exp[(a - 0.5\sigma^2)t + \sigma z_t]}{K + a\, l_0 \int_0^t \exp[(a - 0.5\sigma^2)u + \sigma z_u] du}.$$

For zero volatility, $\sigma = 0$, solutions of the deterministic and the stochastic models are coincided.



The expected rate of growth of the leverage is also the same:

$$\frac{1}{dt}\left\langle \frac{dL_t}{L_t} \right\rangle = a(1 - \frac{1}{K} L_t).$$

Stationary forward Kolmogorov-Fokker-Plank equation

$$-\frac{\partial}{\partial l}[l(a-bl)p(l)]+\frac{1}{2}\frac{\partial^2}{\partial l^2}[\sigma^2 l^2 p(l)]=0$$

can be easily solved for the pdf , $p(l)$, of the random leverage process $L(t)$.

There are two solutions to the stationary Kolmogorov equation (Pascuali, 2001)

A trivial solution:

$p(l)=\delta(l)$

which is a $\delta - Dirac$ distribution. It is associated with the stationary state, $l_1^* = 0$.

A non-trivial solution is a pdf of gamma-distribution of leverage:

$$p(l)=\frac{\beta^\alpha}{\Gamma(\alpha)}l^{\alpha-1}e^{-\beta l}.$$

Gamma distribution is defined for positive parameters of the shape and rate (scale), respectively: $\alpha=\frac{2a}{\sigma^2}-1;\ \beta=\frac{2b}{\sigma^2}$.

Hence it exists on an interval: $0<\sigma^2<2a$.

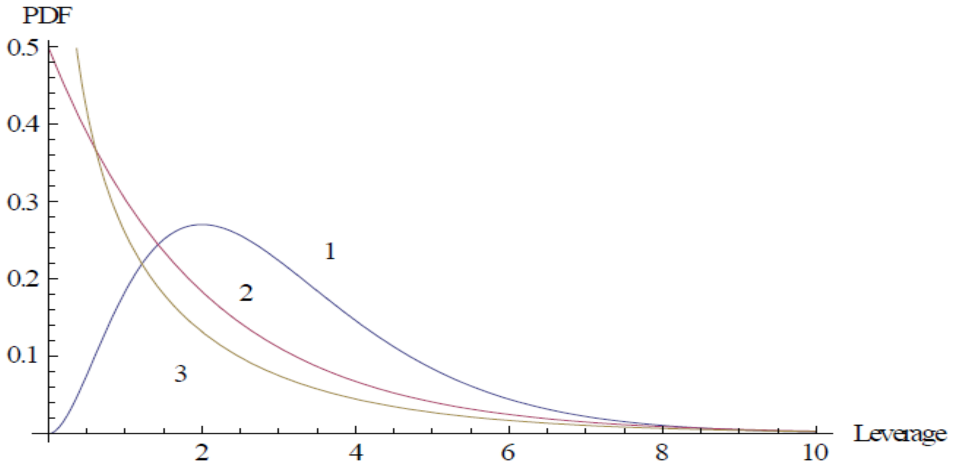
Parameters of the random leverage

a, spread		0.04	
σ, volatility	0.14	0.2	0.22
σ^2, variance	0.02	0.04	0.05
α, shape	3	1	0.6
β, growth rate	1	0.5	0.4
$1/\beta$, scale	1	2	2.5
$\langle \lambda \rangle$ The Lyapunov exponent	-0.02	0	0.01
$\langle L \rangle$, expected value	3	2	1.5
$Mode[L]$	2	0	-

Curve 1: $\sigma^2 - a = 0.02 - 0.04 < 0$ is a peaked gamma distribution of leverage;

Curve 2: $\sigma^2 - a = 0.04 - 0.04 = 0$ is an exponential distribution of leverage;

Curve 3: $\sigma^2 - a = 0.05 - 0.04 > 0$ is a J-shaped gamma distribution of leverage.


PDF
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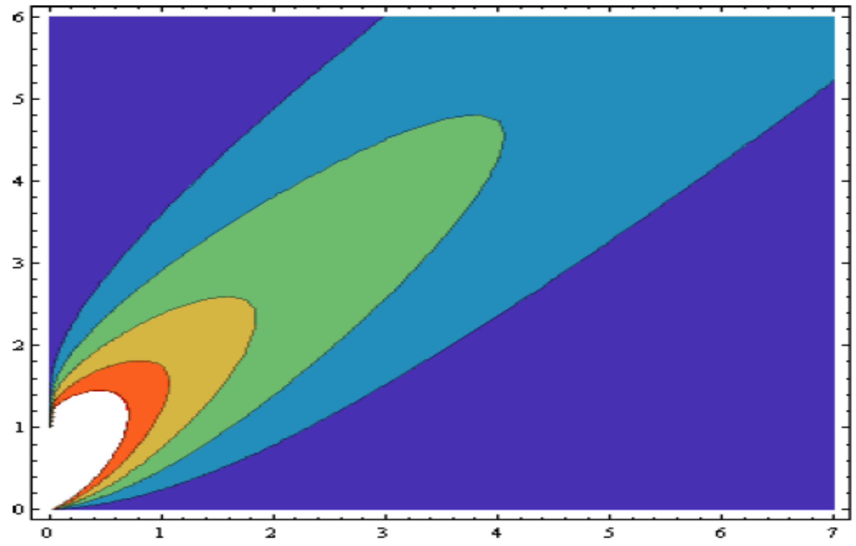
Leverage


6
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7

The long run stability of a stochastic leverage is defined by the ***Lyapunov Exponent***:

$$\langle \lambda \rangle = \frac{\beta^{\alpha}}{\Gamma(\alpha)} \int_0^{\infty} (a - 2bl) l^{\alpha-1} \exp[-\beta l] dl \ .$$

$$\langle \lambda \rangle = \langle a - 2bL \rangle = a - 2b \langle L \rangle = a - 2b\left(\frac{a - \sigma^2}{b}\right) = \sigma^2 - a \ .$$

Since ***"The market can stay irrational longer than you remain solvent"***, according to a popular expression accredited to J.M. Keynes, investors' ***confidence*** in the long run market solvency could be measured as

.

$$\langle \lambda \rangle = (\sigma^2 - a) \begin{Bmatrix} < \\ = \\ > \end{Bmatrix} 0; \quad \begin{matrix} \textit{stability or confidence;} \\ \textit{the system neutrality;} \\ \textit{instability or the loss of confidence.} \end{matrix}$$

The expected value of a gamma-distributed random leverage is

$$\langle L \rangle = \int_0^\infty l\, p(l) dl = \frac{\alpha}{\beta} = K - \frac{\sigma^2}{2b}$$

while its **mode** for $\alpha > 1$ is equal to

$$Mode[L] = \frac{\alpha - 1}{\beta} = K - \frac{\sigma^2}{b}.$$

It is important to notice that **stability of a deterministic system does not imply stability of its stochastic analogue.** Hence the random leverage dynamics could differ substantially from the deterministic one.

Deterministic model is valid	$\sigma^2 \ll a$	K
Stochastic model is stable	$\sigma^2 < a$	$K - \frac{\sigma^2}{b}$
Stochastic model is instable	$a < \sigma^2 < 2a$	$(0, K/2]$
Noise induced chaos	$\sigma^2 > 2a$	0

Failure, Survival and Hazard Rate for the Global Finance

A conceivable answer to these questions could be found via computation of standard hazard, failure and survival functions associated with asymptotical leverage distribution. Considering random leverage as characteristic "lifetime" of a debt system existence its probability density function, $p(l) = \Pr[l < L \leq l + dl]$, would imply information about instantaneous unconditional failures in a process of debt redemption. Hence unconditional probability of a failure to redeem debt outstanding is given by the leverage cumulative distribution, $CDF(l) = \Pr[L \leq l]$. Accordingly, the system would survive if global debt is redeemed with probability $S(l) = 1 - \Pr[L \leq l] = \Pr[L > l]$ for any particular leverage, $l \in [0, \infty)$. Survival function, $S(l)$, as a measure of debt sustainability, is evaluated under boundary condition $S(1) = 1$. Reliability considerations imply, as well, that conditional instantaneous rate of default in the global debt redemption is measured by hazard rate, $h(l)$:

$$h(l) = p(l) / (1 - CDF(l)) \equiv p(l) / S(l) \quad (23) .$$

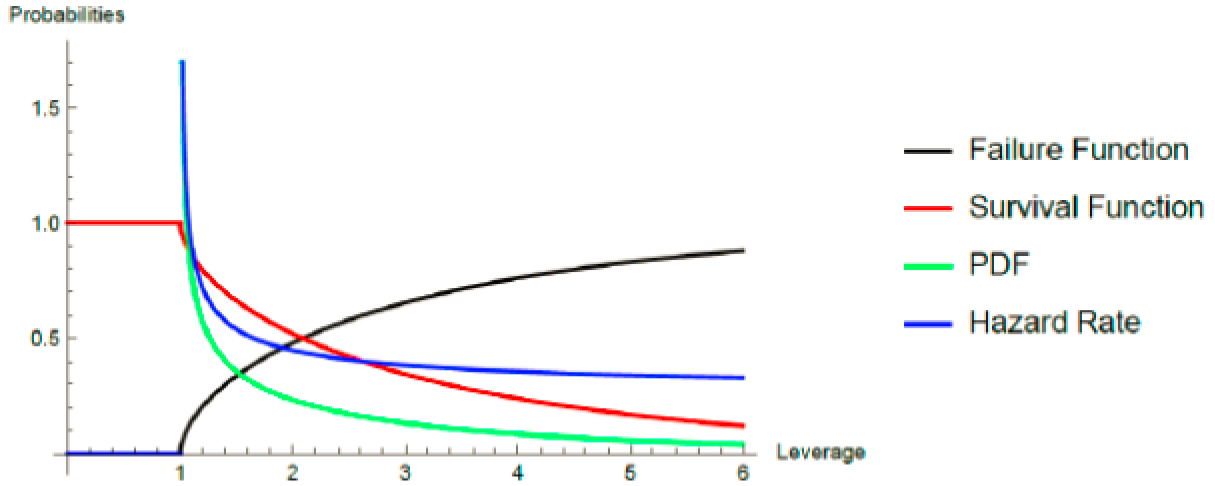


Fig. 7 Failure, survival and hazard functions of stationary leverage distribution[9]

Table 4. Empirical scenario of leverage dynamics

Leverage	$l_1 = 1.062$	$l_N = 2.18*$	$l_2 = 2.45$	$l_3 = 4.0$	$K = 5.27$
Failure rate	5.637	0.215	0.182	0.087	0.053
Hazard rate	5.692	0.436	0.414	0.357	0.338
Failure function	0.0	0.507	0.561	0.756	0.843
Survival function	1.0	0.493	0.439	0.244	0.157

*Parameter l_N is estimated as the mode of stationary gamma distribution.

In the long run aggregate borrowers are assumed taking additional debts contingent on a current weighted average cost of capital, $\mu(1)$, and on expected rate of investment return, r, thus on spread, $a = \mu(1) - r$. In other words, for any viable leverage the expected rate of return on equity, $\rho(l)$, is an indicator of the long run supply of debt:

$$\rho(l) = r + (\mu - r)l;\ \mu = \mu(1) \quad (24).$$

Simultaneously, but independently of borrowers, the collective of debt holders (creditors) are expecting to receive a positive return on assets, given the current rate of return on equity, $\rho = \rho(1)$, and another spread, $c = \rho(1) - r$. Note, that the inverted leverage scale, l^{-1}, preserving measure of debt follows changes in capital, implicitly,[10] and capital is the factor of the debt demand. Hence the long run demand for debt, measured in the inverted leverage scale, is indicated as:

$$\mu(l^{-1}) = r + (\rho - r)l^{-1};\ \rho = \rho(1) \quad (25).$$

$$\mu(l_N^{-1}) = \rho(l_N) \qquad (26).$$

The positive root of equality (26), l_N, by its economic meaning, defines the *anchor leverage*:

$$l_N \equiv K^{0.5} = \left(\frac{a}{b}\right)^{0.5} \qquad (27).$$

$$Mode[L] = l_N \qquad (28).$$

In its turn, equality (28) implies, with necessity, that the expected variance is no larger than its critical value:

$$\sigma_c^2 = a - \sqrt{ab} \qquad (29).$$

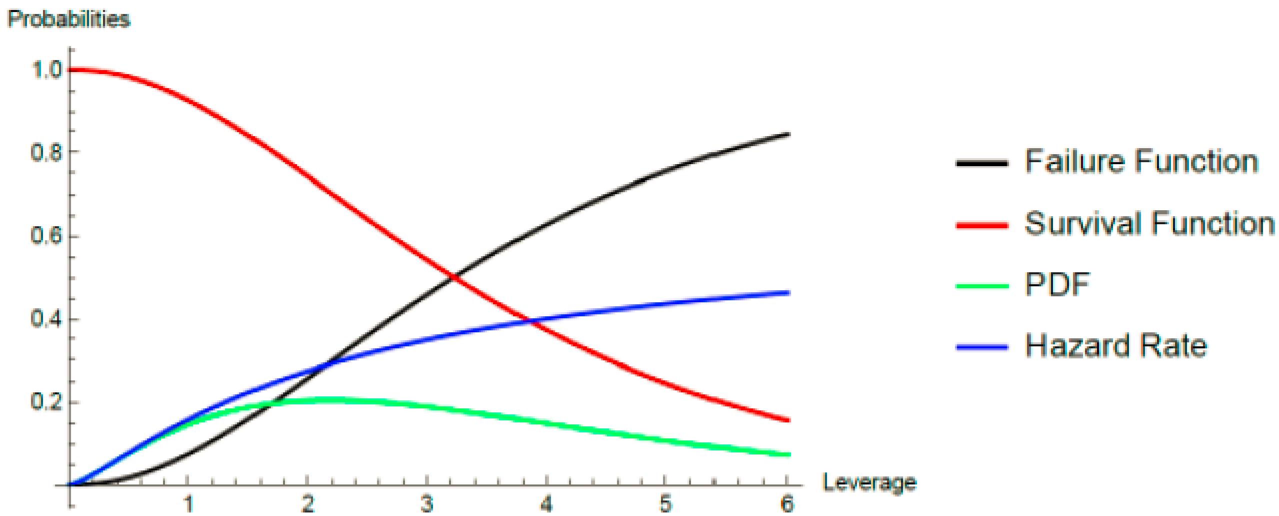


Fig. 8 The peaked gamma distribution of leverage

Table 5. The "optimistic" scenario of leverage dynamics

Leverage	$l_1 = 1.0$	$l_N = 2.18$*	$l_4 = 3.23$	$\langle L \rangle = 3.73$	$K = 5.27$
Failure rate	0.146	0.205	0.181	0.155	0.097
Hazard rate	1.578	0.290	0.362	0.388	0.444
Failure function	0.074	0.294	0.5	0.586	0.782
Survival function	0.926	0.706	0.5	0.414	0.218

*Parameter l_N is estimated as the mode of stationary gamma distribution.

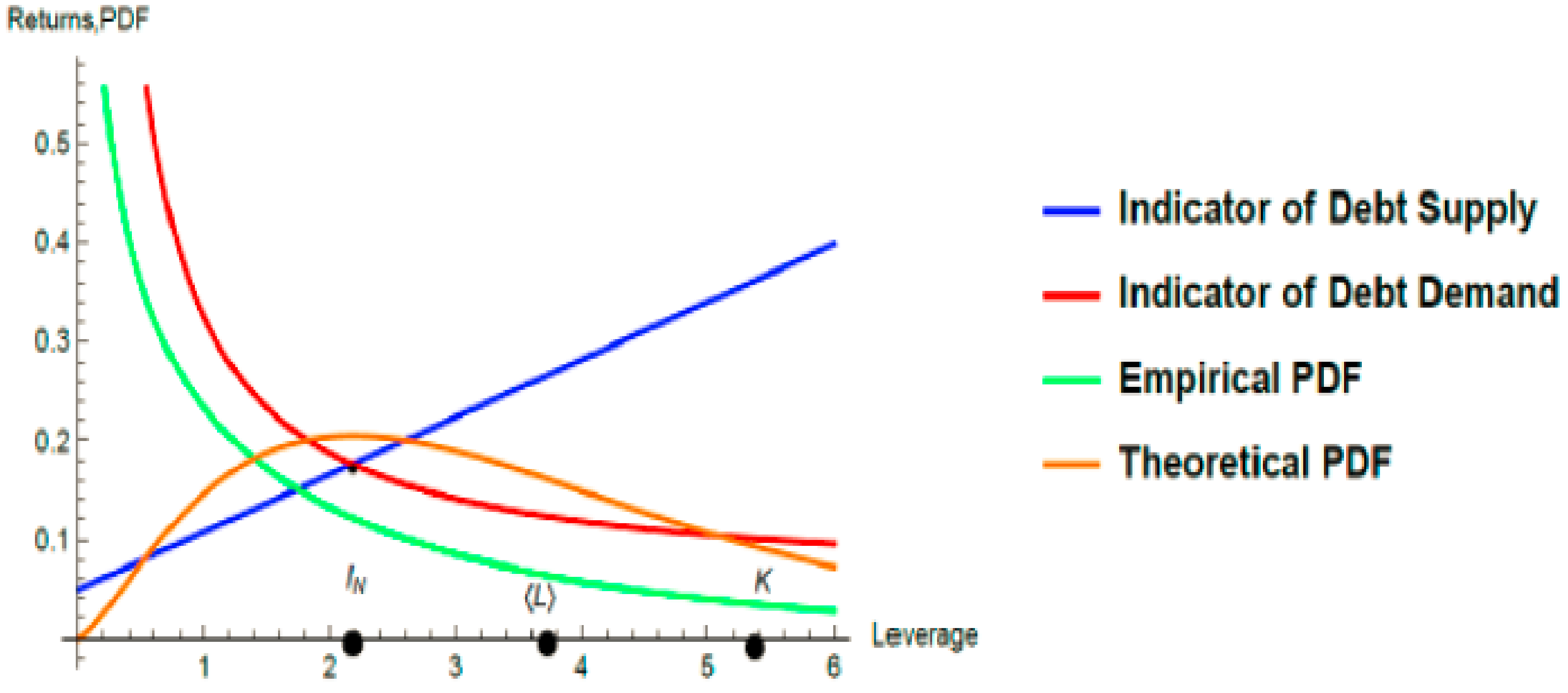


Fig. 9 Gamma distributed leverage and its anchor

Development of financial markets is intertwined with evolution of real markets. As a small step towards a unified macrofinancial-cum-macroeconomic theory we propose to solve this general problem as an estimation of the expected aggregate collateral ratio. Collateral ratio, l_T, is a measure of correspondence between total financial assets and real resources (approximated by the world GDP) that can be decomposed as

$$l_T = l_t * q_t \qquad (30)$$

Table 6. The Global Collateral Ratio components

	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013
l_t	3.97	3.89	4.08	3.74	3.53	6.4	4.92	4.54	5.43	5.12	4.52
q_t	0.87	0.91	0.84	1.05	1.19	0.55	0.82	0.88	0.67	0.73	0.84
l_T	3.45	3.54	3.42	3.93	4.2	3.52	4.03	3.99	3.64	3.74	3.79

world GDP. Formally, this statement is equivalent to the assumption of global equity-to-GDP coefficients, $Q(t)$, following the Ornstein-Ulenbeck stochastic process:

$$Q(t) = 1 + (Q_0 - 1)\exp[-\kappa t] + \sigma \int_0^t \exp[\kappa(u-t)]dW(u) \quad (31)$$

where 1.0 is the long run average[13], κ is the mean reverting parameter, and σ is the diffusion parameter. Some realizations of this process with $Q_0 = 0.87$; $\kappa = 0.5$; $\sigma = 0.055$ are represented in Fig. 10.

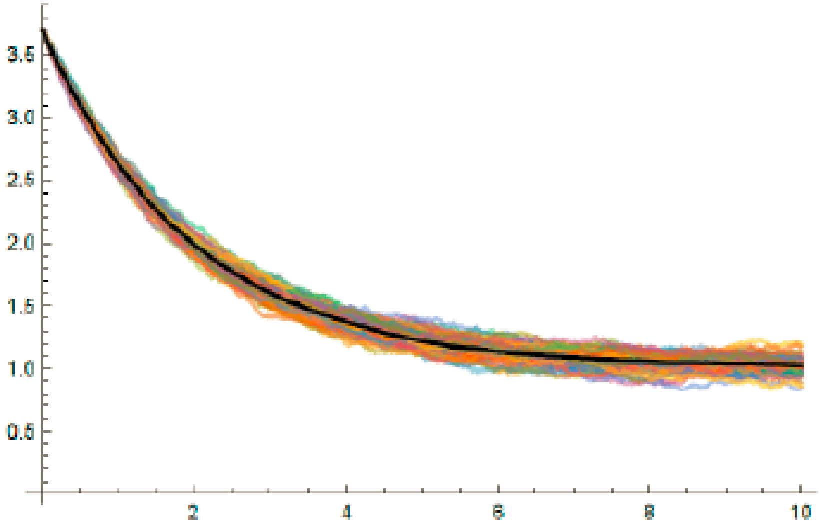
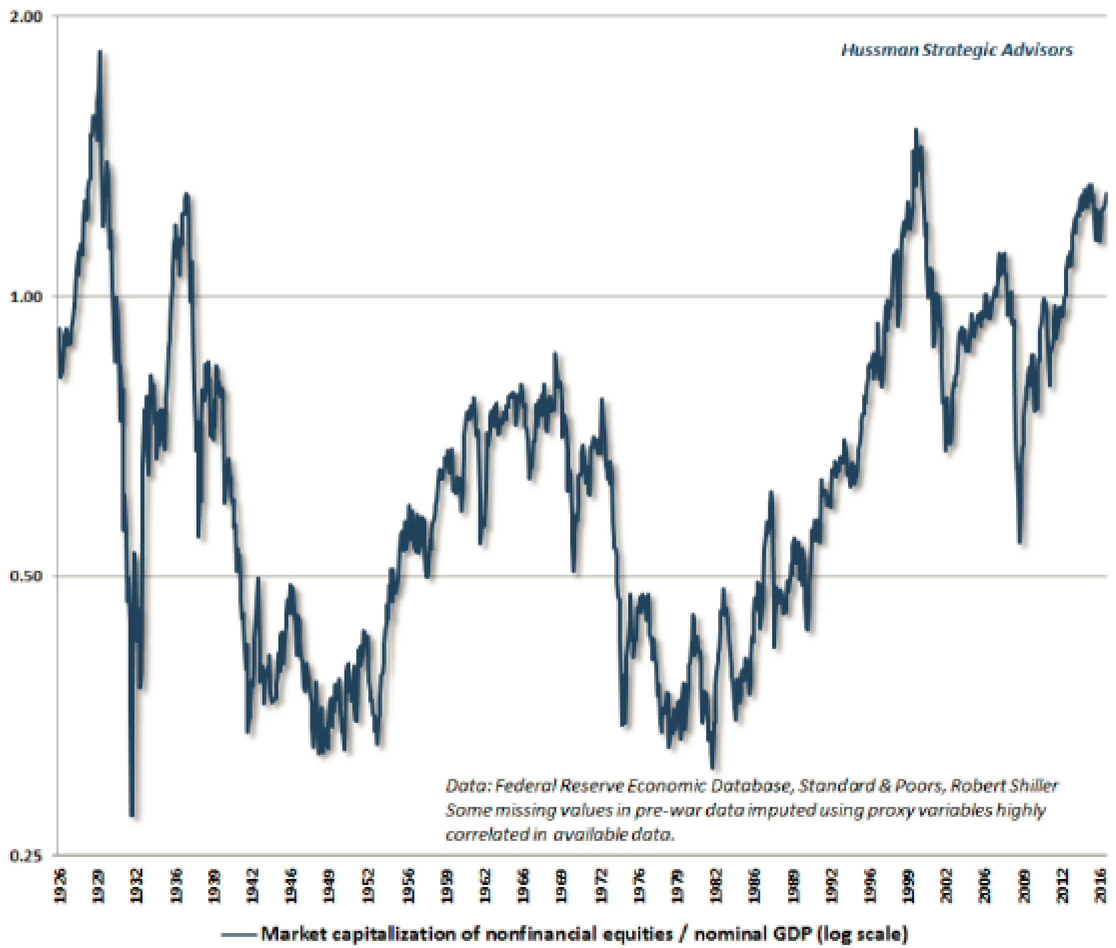


Fig. 10 Global equity-to-GDP realizations

[12] J. Hussman used the ratio of nonfinancial market capitalization to nominal GDP as a measure of long run valuation on the US equity market (Weekly Market Comment, September 12, 2016). His index is, evidently, qualitatively the same as the global equity-to-GDP coefficient, q. In the log scale Hussman's index fluctuated around 1.0 for the last 20 years.



An important asymptotic invariant between capital and debt might be estimated in the global financial market. Note, that collateral ratio (30) has an alternative expression as $l_Y = \frac{A}{x} * \frac{x}{Y}$ where $\frac{A}{x} = \frac{A}{e} * \frac{e}{x} = \frac{l}{l-1}$. Since $l_N = K^{0.5}$ and $\lim_{t \to \infty} \frac{l(t)}{l(t)-1} = \frac{K^{0.5}}{K^{0.5}-1}$, from an asymptotical relation for the collateral ratio, $l_Y(t)$:

$$l_N * [\frac{e}{Y}] = \frac{K^{0.5}}{K^{0.5}-1} * [\frac{x}{Y}] \qquad (32)$$

it follows that

$$[\frac{x}{e}] = K^{0.5} - 1 \qquad (33).$$

Logistic leverage model seems to support conclusions of J. Hussman (*Weekly Market Comments*, 2016) about market speculation fostered by the policy of quantitative easing, QE. The latter, decreasing almost to zero the long term ROI, convinced investors to equate current yields to YTM that disturbed their expectations of future investment returns. Meanwhile expected returns might remain positive only due to overvalued financial assets, hence increased leverage. These processes, as seen in Fig 5, are associated with the upward moving stable leverage (yellow curve), qualitatively similar to trajectory of a discount coupon bond. Under particular conditions it could be made compatible with the lagged convex trajectory of Minsky's bubble (red curve). This is tantamount to acceleration of the Ponzi game among investors, discerned by the short run market internals. Though the latter are absent in our model it explained the long run market tendencies in a way similar to Hussman's analysis.

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Show[p1, p2]
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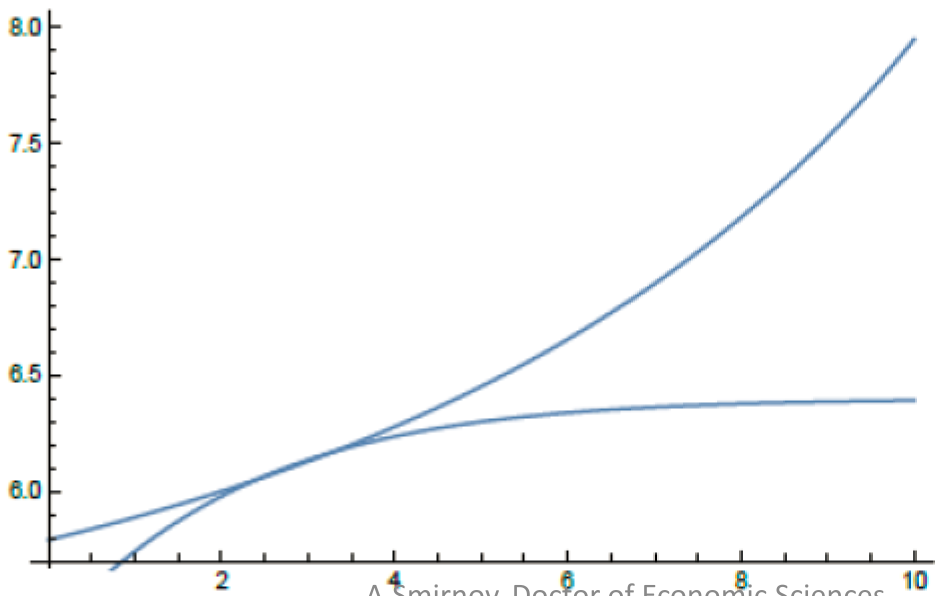
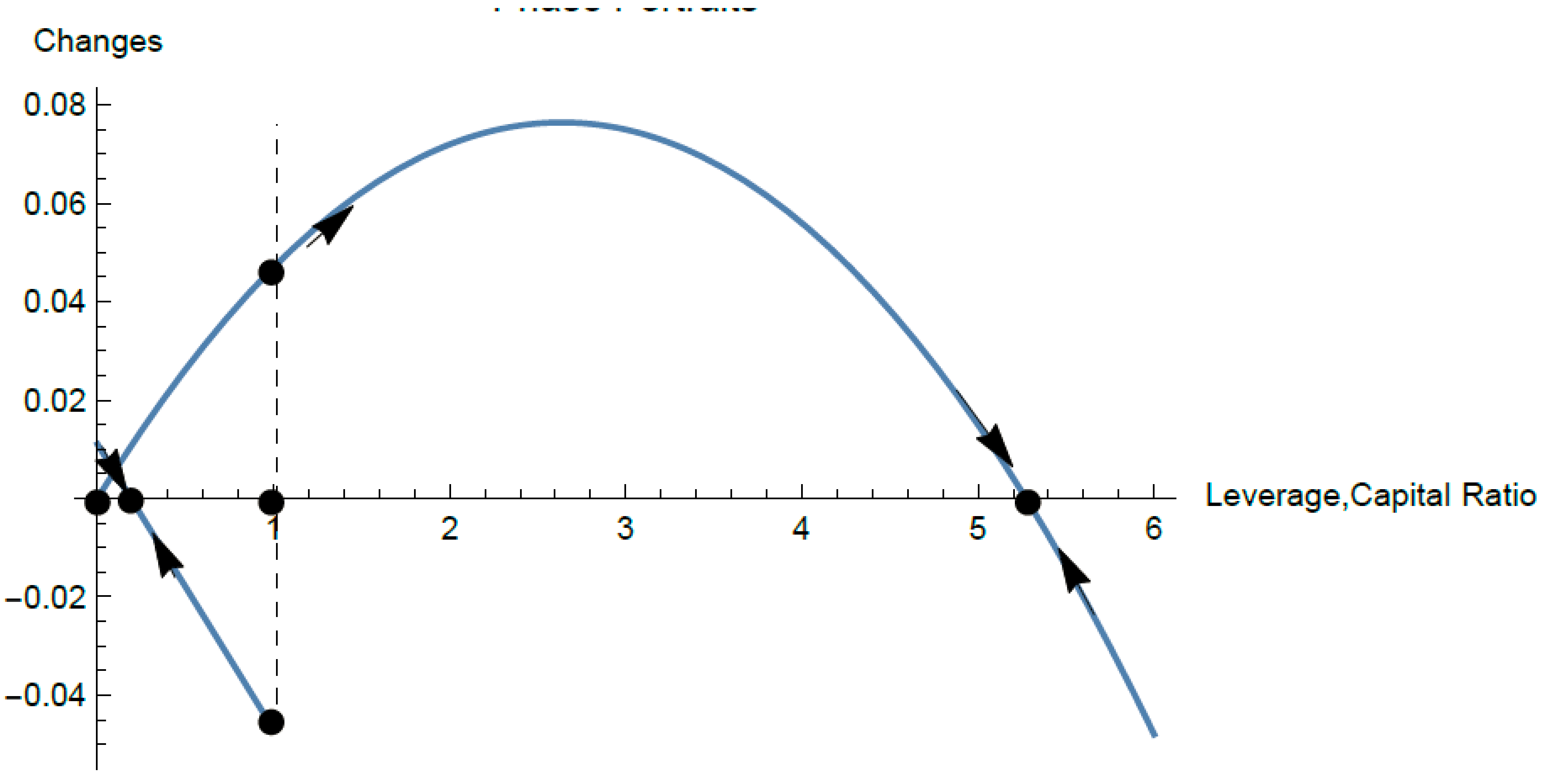


Another interesting aspect of the stochastic logistic model application might be mentioned about briefly. In the paper the leverage volatility was considered constant because of impossibility to differentiate among numerous factors influenced that parameter. On the other hand, successful monetary reforms might lead to transformation of the entire structure of diffusion: namely, linear noise in the rate of change, $\sigma L(t)dW(t)$, would make quadratic the component, $\sigma L^2(t)dW(t)$, in the logistic SDE (18). In that case a divergent deterministic system (with negative spreads) could have been stabilized by adding random noise to the system, as it was rigorously proved (Mao et al, 2002). Though the economic substance of these processes is yet unclear, exploring possibilities of bubbles negation is worthwhile of further research.


Changes
0.08
0.06
0.04
0.02
−0.02
−0.04
Leverage,Capital Ratio
1
2
3
4
5
6